CFAspace

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Academy of Professional Finance 专业金融学院

CFA Level II

Portfolio Management:

Portfolio Concepts

Lecturer: Nan Chen

Framework

Reading	Changes
Reading: Portfolio Concepts	No
Re… urn: The Information Ratio	Added
Reading: The Fundamental Law of Active Management	Altered
Reading: The Portfolio Management Process and the IPS	No

- 1. Mean-Variance Analysis
  - 1.1 Expected Return, Variances and Covariances of an n-asset Portfolio
  - 1.2 Minimum-Variance Frontier and Efficient Frontier
  - 1.3 How correlation and number of assets affect diversification benefits
  - 1.4 CAL, CML and CAPM
  - 1.5 Practical Issues in Mean-Variance Analysis
- 2. Multifactor Models
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  - 2.4 Application of Multifactor Models in Analyzing Sources of Return and Risk

Item Set Example

Joshua White Scenario

(This case was adapted from the CFA mock exams)

Joshua White is an equity portfolio manager for Eastwood Investments, who currently uses the Capital Asset Pricing Model (CAPM) to evaluate securities and mean-variance portfolio optimization to construct equity portfolios. White is meeting to two assistant portfolio managers, Stephen Butler and Deb Miller . Butler and Miller have been asked to do some research on ways to improve on the methods currently being used by Eastwood to evaluate securities and develop portfolios.

White begins the meeting by outlining some issues relating to the CAPM. He makes the following statements:

Statement 1

"One of the reasons I am uncomfortable using the CAPM is that it makes some very restrictive assumptions such as :

*investors pay no taxes on returns and no transaction costs on trades,

*investors have unique views on expected returns, variances and correlations of securities, and

*investors can borrow and lend at the same risk-free rate of interest."

Statement 2

"We are also faced with a problem that our mean-variance optimization models can generate unstable minimum-variance efficient frontiers. Consequently, we face considerable uncertainty regarding recommendations we make to our clients on asset allocation. I attribute the instability to our use of:

•A short sales constraint, and

•Historical betas."

Item Set Example

Butler suggests that multifactor models provide a better way to model stock returns. He states that "there are two ways to model stock returns using the following multifactor model:
$R_i = a_i + b_{i1}F_1 + b_{i2}F_2 + \ldots + b_{ik}F_k + \varepsilon_i$

Model1: In this model, stock returns (R_i) are determined by surprises in economic factors such as GDP growth and the level of interest rates.

Model2: Here, stock returns (R_i) are determined by factors that are company attributes such as PE ratios and market capitalization.

While the interpretation of the intercept a_i is similar for both models, the factor sensitivities b_i are interpreted differently in the two models."

Miller notes that a multifactor Arbitrage Pricing Model (APT) provides a much better basis than the CAPM for calculating expected portfolio returns and evaluating portfolio risk exposures. In order to illustrate the advantages of the multifactor APT model, Miller provides information for two portfolios Eastwood currently manages. The information is provided below in the exhibit. The current risk-free rate is 2%.

Factor Sensitivities and Risk Premia

Risk Factor	Factor Sensitivities			Factor Risk Premium (%)
	Portfolio A	Portfolio B	Benchmark	
Confidence Risk	0.81	0.04	0.5	4.5
Inflation Risk	-0.15	-0.45	-0.25	-1.2
Business Cycle Risk	1.23	0.09	0.9	5.2

Item Set Example

Miller makes the following statement:

Statement 3
"We can tell from the exhibit that Portfolio A is structured in such a manner that it will benefit from an expanding economy and improving confidence because the factor sensitivities for confidence risk and business cycle risk exceed the factor sensitivities for the benchmark. Portfolio B has very low factor sensitivities for confidence risk and business cycle risk but moderately high exposure to inflation risk, therefore Portfolio B can be referred to as a factor portfolio for inflation risk."

White wants to examine how active management is contributing to portfolio performance. Miller responds with the following statement:

Statement 4
"Our models show that Portfolio A has annual tracking error of 1.25% and an information ratio of 1.2 while Portfolio B has an annual tracking error of 0.75% and an information ratio of 0.87."

Item Set Example

1. Which assumption of the CAPM is *most likely* incorrect in White's statement 1? The assumption regarding:
A. Borrowing and lending
B. Taxes and transaction costs
C. Expected returns, variances and correlations

2. Is White's statement 2 most likely correct?
A. Yes.
B. No, she is incorrect about the short sales constraint.
C. No, she is incorrect about the use of historical betas.

3. With regard to the statement on multifactor models, Butler is most likely incorrect with respect to the:
A. Intercept value a_i.
B. Factor sensitivities b_i.
C. Description of the factors.

4. Based on the information in the exhibit, the expected return for portfolio A is closest to:
A. 8.4%
B. 10.2%
C. 12.2%

5. Is Miller's Statement 3 most likely correct?
A. Yes.
B. No, she is incorrect about Portfolio A.
C. No, she is incorrect about Portfolio B.

6. Based on Statement 4 by Miller, an appropriate conclusion is that the portfolio that has benefited the most from active management is:
A. Portfolio B because of tracking error.
B. Portfolio A because of the information ratio.
C. Portfolio B because of the information ratio.

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1.1 Expected Return, Variances and Covariances of an n-asset Portfolio

Mean-Variance Analysis:

the use of expected returns, variances, and covariances of individual investments to analyze the risk-return tradeoff of combinations (i.e., portfolios) of these assets.

Assumptions of Mean-Variance Analysis:

- All investors are risk averse.
- Expected returns, variances, and covariances are known for all assets.
- Investors create optimal portfolios by relying solely on expected returns, variances and covariances. No other distributional parameter is used.
- Investors face no taxes or transaction costs.

1.1 Expected Return, Variances and Covariances of an n-asset Portfolio

For a Two-Asset Portfolio:

Portfolio Expected Return:

$$\boxed{E(R_P) = w_1 E(R_1) + w_2 E(R_2)}$$

where:
$E(R_P)$ = expected return on Portfolio P
w_i = proportion ("weight") of the portfolio allocated to Asset i
$E(R_i)$ = expected return on Asset i

Portfolio Variance:

$$\rho_{1,2} = \frac{Cov_{1,2}}{\sigma_1 \sigma_2}$$

$$\boxed{\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}} \longrightarrow \sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

where:
σ_P^2 = variance of the returns for Portfolio P
σ_1^2 = variance of the returns for Asset 1
σ_2^2 = variance of the returns for Asset 2
w_i = proportion (weight) of the portfolio allocated to Asset i
$Cov_{1,2}$ = covariance between the returns of the two assets

For a Three-Asset Portfolio:

Portfolio Expected Return:

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3)$$

Portfolio Variance:

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \left[w_3^2\sigma_3^2\right] + 2w_1w_2\,cov_{1,2} + \left[2w_1w_3\,cov_{1,3} + 2w_2w_3\,cov_{2,3}\right]$$

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2 + 2w_1w_3\rho_{1,3}\sigma_1\sigma_3 + 2w_2w_3\rho_{2,3}\sigma_2\sigma_3$$

$$\sigma_P = \sqrt{\sigma_P^2}$$

1.1 Expected Return, Variances and Covariances of an n-asset Portfolio

EX: Calculate the expected return and standard deviation of the following three-asset portfolio:

	A	B	C
Weight	0.40	0.25	0.35
Expected Return	11%	25%	30%
Standard Deviation	15%	20%	25%
	Correlations:		
A and B	0.30		
A and C	0.10		
B and C	0.50		

Ans:

- $\boldsymbol{E(R_p)} = w_A E(R_A) + w_B E(R_B) + w_C E(R_C)$

$= (0.40)(0.11)+(0.25)(0.25)+(0.35)(0.30)$

$=21.15\%$

- ***Portfolio Variance***

$= (0.40)^2(0.15)^2 + (0.25)^2(0.20)^2 + (0.35)^2(0.25)^2 + 2(0.40)(0.25)(0.30)(0.15)(0.20)$
$+2(0.40)(0.35)(0.10)(0.15)(0.25) + 2(0.25)(0.35)(0.50)(0.20)(0.25)$

$=0.02098$

Portfolio Standard Deviation $= (0.02098)^{1/2} = 14.485\%$

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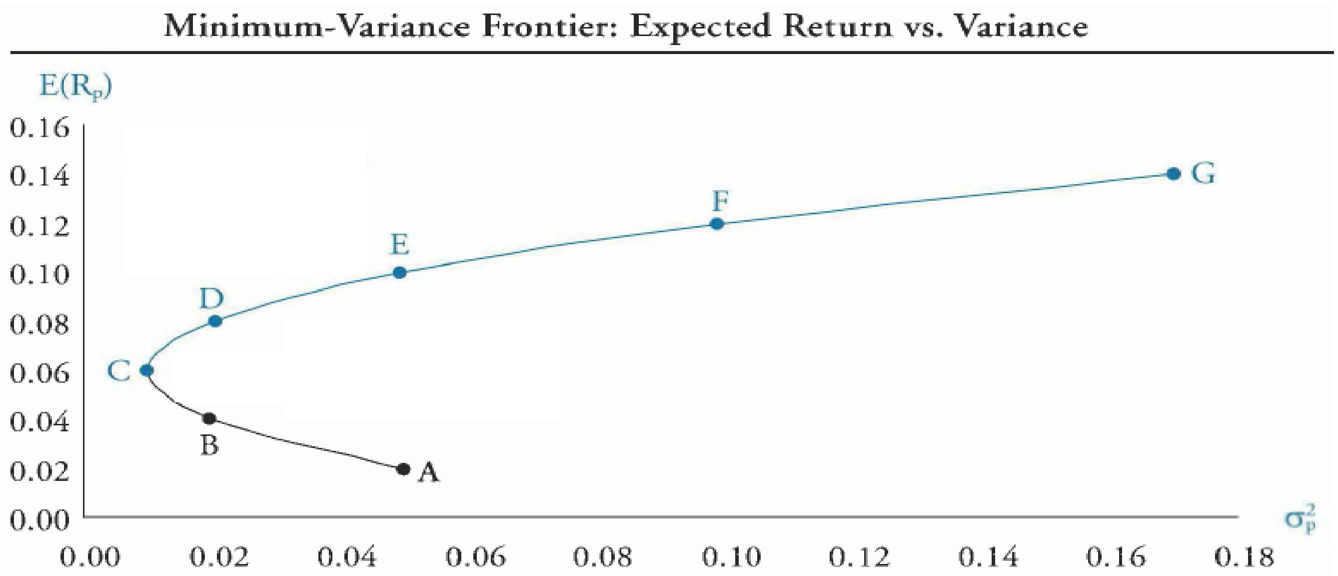
1.2 Minimum-Variance Frontier and Efficient Frontier

- *A **Minimum-Variance Portfolio*** has the ***smallest*** variance among all portfolios with ***identical*** expected return.

- *The **Minimum-Variance Frontier*** is a graph of the expected return/variance combinations for all minimum-variance portfolios.



- *The **Efficient Frontier*** is a plot of efficient portfolios that have
  - ✓ Minimum risk of all portfolio with the same expected return.
  - ✓ Maximum expected return for all portfolios with the same risk.

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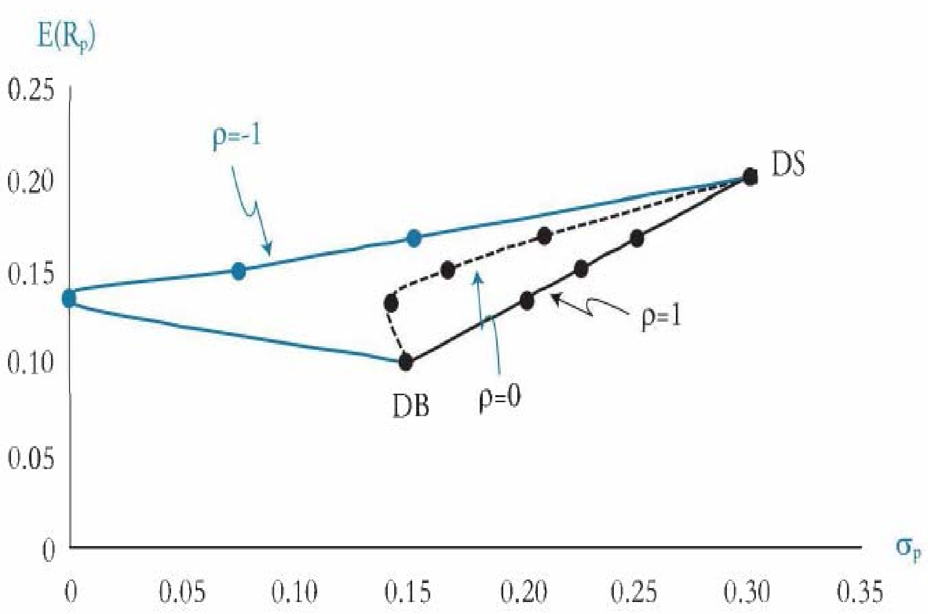
1.3 How Correlation and Number of Assets Affect Diversification Benefits

1.3.1 Correlations between assets: lower correlation means greater diversification benefits.

	Expected Return	Standard Deviation
Domestic Stock	0.20	0.30
Domestic Bond	0.10	0.15

$$E(R_P) = w_1 E(R_1) + w_2 E(R_2)$$
$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

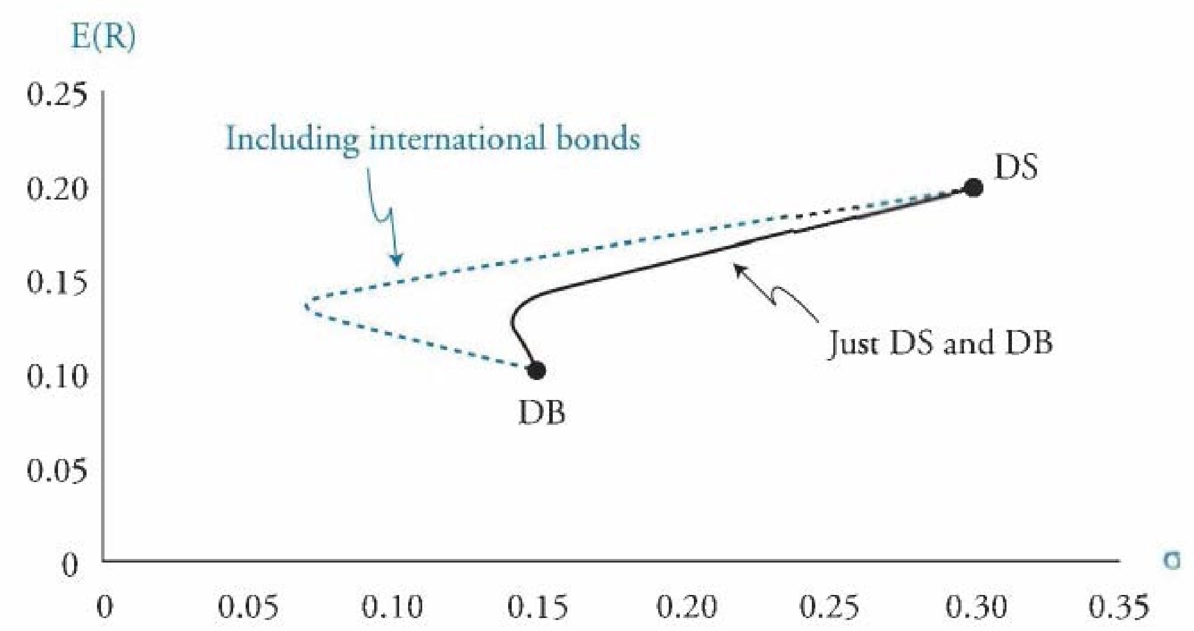
Correlation	*DS % Allocation*	*DB % Allocation*	$E(R_p)$	σ_P
+1	100.00	0.00	0.200	0.300
	66.67	33.33	0.167	0.250
	50.00	50.00	0.150	0.225
	33.33	66.67	0.133	0.200
	0.00	100.00	0.100	0.150
0	100.00	0.00	0.200	0.300
	66.67	33.33	0.167	0.206
	50.00	50.00	0.150	0.168
	33.33	66.67	0.133	0.141
	0.00	100.00	0.100	0.150
−1	100.00	0.00	0.200	0.300
	66.67	33.33	0.167	0.150
	50.00	50.00	0.150	0.075
	33.33	66.67	0.133	0.000
	0.00	100.00	0.100	0.150



1.3 How Correlation and Number of Assets Affect Diversification Benefits

1.3.2 Number of assets included in the portfolio: more assets mean greater diversification benefits.

	Expected Return	Standard Deviation
Domestic Stock	0.20	0.30
Domestic Bond	0.10	0.15
International Bond	*Correlation between IB and DS <1* *Correlation between IB and DB <1*	



1.3 How Correlation and Number of Assets Affect Diversification Benefits

➤ ***Variance of an Equally-Weighted Portfolio of n assets:***

$$\sigma_P^2 = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{Cov}$$

where:

$\overline{\sigma_i^2}$ = average variance of all assets in the portfolio

$\overline{\text{Cov}}$ = average covariance of all pairings of assets in the portfolio

Conclusion 1: *The variance of an equally-weighted portfolio approaches the average covariance as n gets large.*

$$\sigma_P^2 = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{Cov} = \frac{1}{n}\overline{\sigma_i^2} + \frac{n-1}{n}\overline{\sigma_i^2}\rho = \overline{\sigma_i^2}\left(\frac{1-\rho}{n} + \rho\right)$$

Conclusion 2: *As n gets very large, the maximum amount of risk reduction occurs:* $\sigma_P^2 \approx \overline{\sigma_i^2}(\rho)$

Conclusion 3: *The lower the level of correlation, the greater the potential diversification benefits, but the greater the number of securities required to realize them.*

Thank You!