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Academy of Professional Finance 专业金融学院

CFA Level II

Fixed Income:

The Term Structure and Interest Rate Dynamics

Lecturer: Nan Chen

Framework

Reading

Reading: The Term Structure and Interest Rate Dynamics

- Spot Rates, Forward Rates, Yield Curve
  - Relationships among spot rates, forward rates, YTM and expected and realized return on bonds
  - Forward pricing and forward rate models
  - Evolution of spot rates in relation to forward rates
  - Strategy of riding the yield curve
  - Swap rate curve
- Swap Spread, I-Spread, TED Spread, LIBOR-OIS Spread
- Traditional Theories of Term Structure of Interest Rates
- Modern Term Structure Models
- Measuring Yield Curve Risk
- Term Structure of Interest Rate Volatility

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Item Set Example

Eastwood Scenario

(This case was adapted from the CFA mock exams)

Joshua White, CFA, and Stephen Butler are co-managers of Eastwood Core Bond Fund. Eastwood is a fixed income fund that is benchmarked against the U.S. Barclays Aggregate Bond Index. The fund and index contain securities in the Treasury, credit, asset-backed, and mortgage-backed sectors of the market.

White and Butler first discuss their expectations on the direction of interest rates. White states:" Rates are attractive across the curve. The 7- to 10-year part of the curve looks expensive, but that should not deter us because it is driven by insurance companies hedging their liabilities." Butler responds:" Interest rates for long maturity bonds look attractive; the risk premium appears to compensate us for the potential downside of adding duration. This premium is above the expected forward rates."

White then discusses yield spread with Deb Miller, CFA, Eastwood's corporate bond analyst. Miller offers the following observations:

Observation1:If the three-month T-bill rate drops and the Libor rate remains the same, the relevant TED spread increases.

Observation2:I-spread not only reflects time value, but also compensation for credit and liquidity risk.

Observation3: Given the yield curve for US Treasury zero-coupon bonds, Z-spread is more helpful pricing a corporate bond than TED spread and Libor-OIS spread.

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Item Set Example

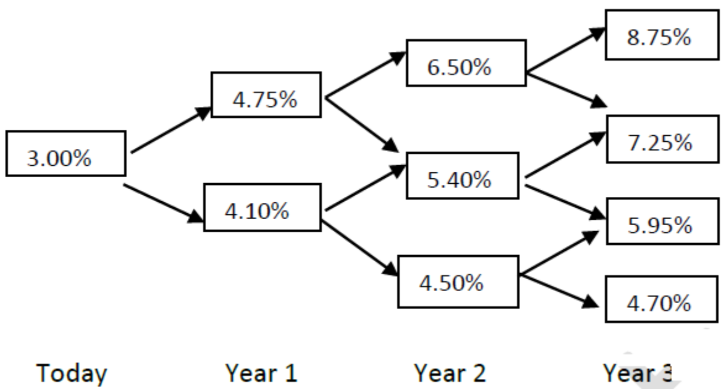
Butler then focuses on the mortgage securities in the portfolio. He asks Miller to explain what the cash flow implications are for a pool of mortgages in the portfolio. Butler describes the mortgages in the pool as having a 20-month average age, and the pool has a monthly mortality of 0.4353%.

Miller then offers to go over her valuation of a callable bond issue by a company she has been researching. The bond is callable at $101.50 every year starting one year from today. She uses the data in Exhibit 1 for her valuation.

Exhibit 1

Binomial Interest Rate Tree (9% Volatility Assumed)

for Valuing a $100 Par Value 3-Year Callable Bond with a 6.25% Coupon



Butler tests Miller's knowledge of securitized transactions by asking her to explain the tranches of the ABS securitization in Exhibit 2.

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Item Set Example

Exhibit 2 ABS Structure

Bond Class	Par Value ($millions)
A1 (senior)	40
A2 (senior)	25
A3 (senior)	20
B (subordinate)	8
C (subordinate)	7
Total	**100**

Miller provides the following explanations:” This securitization is a sequential-pay transaction. As such, interest payments are paid to each bond class periodically. Principal repayments are applied first to the lowest tranche, in this case tranche C, to protect investors from prepayment risk. The senior-subordinate structure has been established for credit tranching to protect against defaults, with subordinated tranches sharing equally in any losses”.

White then asks Miller which of the various valuation models would be most appropriate for assessing relative value. Costas responds:” It really depends on the characteristics of the security. As examples, consider the following three securities.”

Security A: 5%, non-callable 30-year corporate bond selling at a discount.
Security B: 4%, 20-year Ginnie Mae debenture callable in five years.
Security C: Zero-coupon, 10-year Treasury bond.

Miller explains that the most appropriate measure to use are a zero-volatility spread for Security A, an option-adjusted spread (OAD) for Security B, and a nominal spread for Security C.

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Item Set Example

1. Which theory of the term structure of interest rates least likely explains the views of either White or Butler
A. Preferred habitat
B. Pure expectations
C. Liquidity preference

2. In observing yield spread, Miller is least likely correct respect to:
A. Observation 1
B. Observation 2
C. Observation 3

3. The prepayment estimate of the mortgage pool Butler describes is closest to a PSA of :
A. 85%
B. 128%
C. 131%

4. Using the data in Exhibit 1, the current value of the callable bond Miller is analyzing is closes to:

A. 105.56
B. 104.61
C. 101.40

5. Miller's explanation of the securitization in Exhibit 2 is least likely correct with respect to:
A. Interest payments and losses
B. Losses and principal payments
C. Interest and principal payments

6. Miller is least likely correct with respect to the valuation measure for:

A. Security A
B. Security B
C. Security C

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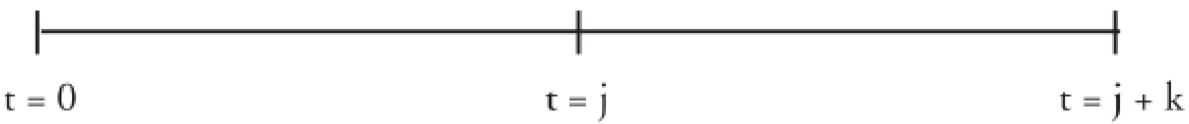
Relationships among spot rates, forward rates, YTM and expected and realized return on bonds

Spot Rates

$$P_T = \frac{1}{(1+S_T)^T}$$

- P_T: discount factor
- S_T: spot rate for maturity T
- Spot (Yield) Curve: the term structure of spot rates-the graph of the spot rate S_T versus the maturity T.
- Spot Rate: an interest rate for a loan initiated today.

Forward Rates



- Forward Curve: the term structure of forward rates-the graph of the forward rates versus the maturity.
- Forward Rate: an interest rate that is determined today for a loan that will be initiated in a future time period.

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Relationships among spot rates, forward rates, YTM and expected and realized return on bonds

YTM=Expected Return=Realized Return when…

$$\text{Bond Price} = \frac{\text{Cashflow 1}}{(1+\text{yield})^1} + \frac{\text{Cashflow 2}}{(1+\text{yield})^2} + \dots + \frac{\text{Last Cashflow}}{(1+\text{yield})^n}$$

all three of the following are true:

-The bond is held to maturity;

-All payments (coupon and principal) are made on time and in full;

-All coupons are reinvested at the original YTM.

YTM and Spot Rate

- For a zero-coupon bond, YTM=Spot Rate
- For a coupon bond, if the spot rate curve is not flat, the YTM will not be the same as the spot rate.

EX: Compare the YTM and spot rates for a 3-year, 4% annual-pay, \$1,000 face value bond given the following spot rate curve: S_1=5%, S_2=6% and S_3=7%.

1. Calculate the price of the bond using the spot rate curve:

$$\text{Price} = \frac{40}{(1.05)} + \frac{40}{(1.06)^2} + \frac{1040}{(1.07)^3} = \$922.64$$

2. Calculate the yield to maturity (y_3):

N = 3; PV = –922.64; PMT = 40; FV = 1,000; CPT I/Y → 6.94

y_3= 6.94%

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Forward Pricing Model

EX: Investor A purchases a $1 par value, zero-coupon bond maturing in 5 years. Investor B enters into a 2-year forward contract to purchase a $1 par value, zero-coupon bond maturing in 3 years.

Given the 2-year spot rate S_2=4% and 5-year spot rate S_5=6%, calculate the price Investor B agrees to pay in 2 years, for the 3-year bond, i.e. the forward price of the 3-year bond in 2 years.

Ans: Arbitrage-Free Pricing: Because the $1 cash flows at year 5 are the same, these two investments should have the same price today:

Investor A: $P_5 = \$1/(1+S_5)^5 = \$1/(1+6\%)^5$

$=$

Investor B: $F(2,3)/(1+S_2)^2 = F(2,3)/(1+4\%)^2$

F(2,3)=$0.8082, the price agreed today, to pay in 2 years, for a 3- year bond that will pay $1 at maturity, i.e. the forward price of a 3 year bond in 2 years

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Forward Pricing and Forward Rate Models

Forward Rate Model

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j \, [1 + f(j,k)]^k$$

EX: Given the 2-year and 5-year spot rates are S_2=4% and S_5=6%, calculate the implied 3-year forward rate for a loan starting 2 years from now.

Ans:

$$[1 + f(j,k)]^k = [1 + S_{(j+k)}]^{(j+k)} / (1 + S_j)^j$$

$$[1 + f(2,3)]^3 = [1 + 0.06]^5 / [1 + 0.04]^2$$

$$f(2,3) = 7.35\%$$

Implication: The forward rate f(2,3) should make investors indifferent between

buying a 5-year zero-coupon bond
v.s.
buying a 2-year zero-coupon bond and at maturity reinvesting the principal for 3 additional years.

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Evolution of Spot Rates in Relation to Forward Rates

$$[1 + S_{(j+k)}]^{(j+k)} = (1 + S_j)^j\,[1 + f(j,k)]^k$$

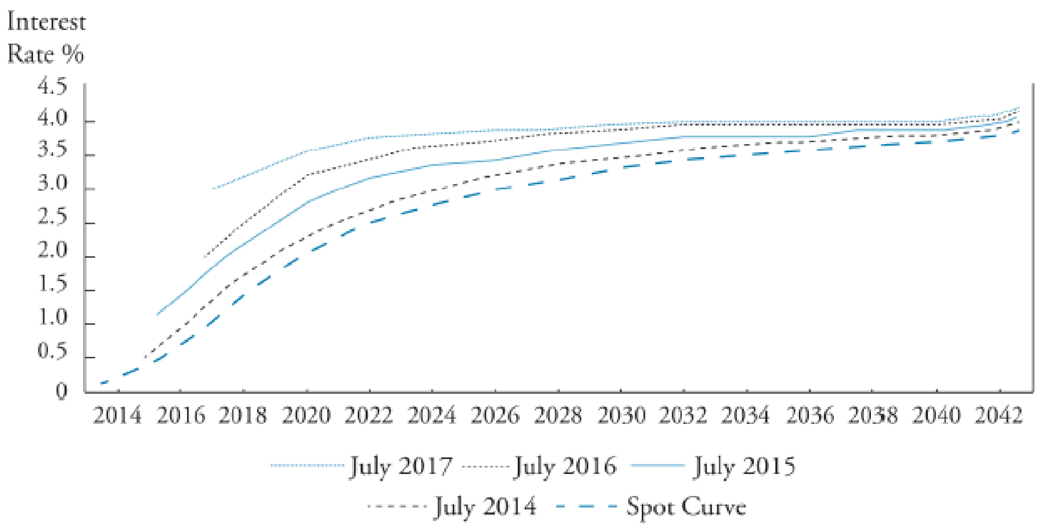
For an upward-sloping spot curve,

- The forward rate rises as j increases;
- The forward curve will be above the spot curve.

For a downward-sloping spot curve,

- The forward rate declines as j increases;
- The forward curve will be below the spot curve.

Figure 1: Spot Curve and Forward Curves



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Evolution of Spot Rates in Relation to Forward Rates

EX: The 1-year, 2-year and 3-year benchmark spot rates are 3%, 4% and 5%. Compute the 1-year holding period return of

1. 1-year, zero-coupon bond.
2. 2-year, zero-coupon bond.
3. 3-year, zero-coupon bond.

- $[1+S_{(j+k)}]^{(j+k)} = (1+S_j)^j\,[1+f(j,k)]^k$ ⇨

$$[1+f(1,1)]^1 = \frac{(1+S_2)^2}{(1+S_1)} = \frac{(1.04)^2}{(1.03)} \rightarrow f(1,1) = 0.0501 \text{ and}$$

$$[1+f(1,2)]^2 = \frac{(1+S_3)^3}{(1+S_1)} = \frac{(1.05)^3}{(1.03)} \rightarrow f(1,2) = 0.0601$$

- Holding Period Return = (End. Price −Beg. Price)/Beg. Price
- If the future spot rates actually evolve as forecasted by the forward curve…

1-year holding period return of 1-year, zero-coupon bond=(1-0.9709)/0.9709=3%

- End. Price=$1
- Beg. Price=$1/(1+$S_1$)=$1/(1+3%)=$0.9709

1-year holding period return of 2-year, zero-coupon bond=(0.9523-0.9246)/0.9246=3%

- End. Price=$1/[1+f(1,1)]=$1/(1+0.0501)=0.9523
- Beg. Price=$1/(1+$S_2$)^2=$1/(1+4%)^2=0.9246

1-year holding period return of 3-year, zero-coupon bond=(0.8898-0.8638)/0.8638=3%

- End. Price=$1/[1+f(1,2)]^2=$1/(1+0.00601)^2=0.8898
- Beg. Price=$1/(1+$S_3$)^3=$1/(1+5%)^3=0.8638

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Evolution of Spot Rates in Relation to Forward Rates

EX: The 1-year, 2-year and 3-year benchmark spot rates are 3%, 4% and 5%. Compute the 1-year holding period return of

1. 1-year, zero-coupon bond.
2. 2-year, zero-coupon bond.
3. 3-year, zero-coupon bond.

1-year holding period return of 1-year, zero-coupon bond=(1-0.9709)/0.9709=3%

1-year holding period return of 2-year, zero-coupon bond=(0.9523-0.9246)/0.9246=3%

1-year holding period return of 3-year, zero-coupon bond=(0.8898-0.8638)/0.8638=3%

- If spot rates *evolve as predicted* by forward rates, bonds of all maturities will realize one-period return equal to the one-period spot rate.

- Active bond portfolio management is built on the pre-assumption that the current forward curve may *not accurately predict* future spot rates.

- If an investor believes that future spot rates will be *lower than* corresponding forward rates, then the investor will purchase bonds because the market appears to be discounting the future cash flows at "too high" of a discount rate.

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Bootstrapping

- The par curve represents the yields to maturity on coupon-paying government bonds, priced at par, over a range of maturities.

- Bootstrapping: The zero-coupon rates are determined by using the par yields and solving for the zero-coupon rates one by one, in order from earliest to latest maturities.

- EX: Bootstrap zero-coupon rates from the following rates: One-year par rate = 5%, Two-year par rate = 5.97%， Three-year par rate = 6.91%, Four-year par rate = 7.81%.

-

One-year spot rate = One-year zero-coupon rate = One-year par rate = 5%

$$1 = \frac{0.0597}{(1.05)} + \frac{1 + 0.0597}{[1 + r(2)]^2}$$ => Two-year spot rate=6%

$$1 = \frac{0.0691}{(1.05)} + \frac{0.0691}{(1.06)^2} + \frac{1 + 0.0691}{[1 + r(3)]^3}$$ =>Three-year spot rate = 7%

$$1 = \frac{0.0781}{(1.05)} + \frac{0.0781}{(1.06)^2} + \frac{0.0781}{(1.07)^3} + \frac{1 + 0.0781}{[1 + r(4)]^4}$$ => Four-year spot rate =8%

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Strategy of Riding the Yield Curve

Figure 2: Price of a 3%, Annual Pay Bond

Maturity	*Yield*	*Price*
5	3	100
10	3.5	95.84
15	4	88.88
20	4.5	80.49
25	5	71.81
30	5.5	63.67

- Assumption 1: This is an upward sloping yield curve.
- Assumption 2: The yield curve remains unchanged over the investment horizon.
- An investor with an investment horizon of 5 years.

Maturity Matching	Riding the Yield Curve
Strategy: Purchase a 5-year bond for $100 (par)	Strategy: Purchase a 30-year bond for $63.67, Hold it for 5 years, Sell it for $71.81
Coupon earned: 5*$3	Coupon earned: 5*$3
Capital gains: none	Capital gains: $71.81-$63.67

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Swap Rate Curve

- **Swap (Fixed) Rate**: the fixed rate in an interest rate swap.

- **Swap Rate Curve**: the graph of how swap rates vary for various maturities.

- The **LIBOR swap curve** is arguably the most commonly used interest rate curve.

- As a benchmark interest rate curve, **why swap rate curve is preferred** rather than a government bond yield curve?

  - ✓ Swap rates reflect the credit risk of *commercial banks* rather than governments;
  - ✓ The swap market is not regulated by any government, which makes swap rates in different countries more *comparable*.
  - ✓ The swap curve typically has yield *quotes at many maturities*, while US government bond yield curve has on-the-run issues trading at only a small number of maturities.

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Swap Spread, I-Spread, Z-Spread, TED Spread, LIBOR-OIS Spread

Swap Spread

$$\text{swap spread}_t = \text{swap rate}_t - \text{Treasury yield}_t$$

I-Spread

- I-spread= yield on the risky bond$_t$ – swap rate$_t$

EX: 6% ABC bonds are currently yielding 2.35% and mature in 1.6 years. From the provided swap curve, compute the I-spread.

Tenor	Swap Rate
0.5	1.00%
1	1.25%
1.5	1.35%
2	1.50%

Ans:

Linear interpolation:

$$1.6 \text{ year swap rate} =$$

$$1.5\text{-year swap rate} + \frac{0.10\ (2\text{-year swap rate} - 1.5\text{-year swap rate})}{0.50}$$

$$= 1.35 + \frac{0.10\ (1.50 - 1.35)}{0.50} = 1.38\%$$

I-spread = yield on the bond − swap rate

$$= 2.35 - 1.38 = 0.62\% \text{ or } 62\text{bps}$$

- I-spread only reflects compensation for credit risk and liquidity risk.

Z-Spread

- Z-spread is the spread that, when added to each spot rate on the default-free spot curves, makes the present value of a bond’s cash flows equal to the bond’s market price.

EX: Consider a 3-year, 9% annual coupon corporate bond trading at $89.464. 1-, 2-, 3- year spot rates on Treasuries are 4%, 8.167%, and 12.377%. Compute the zero-volatility spread of the corporate bond.

$$89.464 = \frac{9}{(1.04+ZS)^1} + \frac{9}{(1.08167+ZS)^2} + \frac{109}{(1.12377+ZS)^3} \Rightarrow$$

$$ZS = 1.67\% \text{ or } 167 \text{ basis points}$$

TED Spread

TED spread = (3-month LIBOR rate) – (3-month T-bill rate)

- TED spread is seen as an indication of the risk of interbank loans.

Libor-OIS Spread

- LIBOR-OIS spread= LIBOR rate$_t$ – Overnight Indexed Swap rate$_t$
- LIBOR-OIS spread is a useful measure of credit risk and an indication of the overall wellbeing of the banking system.

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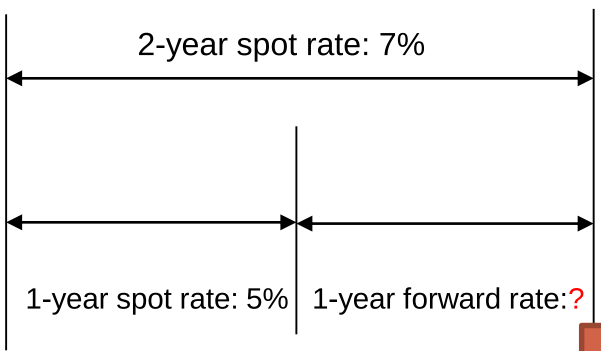
Unbiased Expectation Theory

➢*Interpretation of Forward Rates*

Forward rates are an unbiased predictor of future spot rates. Long-term interest rates equal the mean of future expected short-term rates.

$$1 + 2yr\ spot\ rate = \sqrt{(1 + 1yr\ spot\ rate)(1 + 1yr\ forward\ rate\ in\ 1\ year)}$$



$$1yr\ forward\ rate\ in\ 1\ year$$

$$= \frac{(1+2yr\ spot\ rate)^2}{(1+1yr\ spot\ rate)} - 1$$

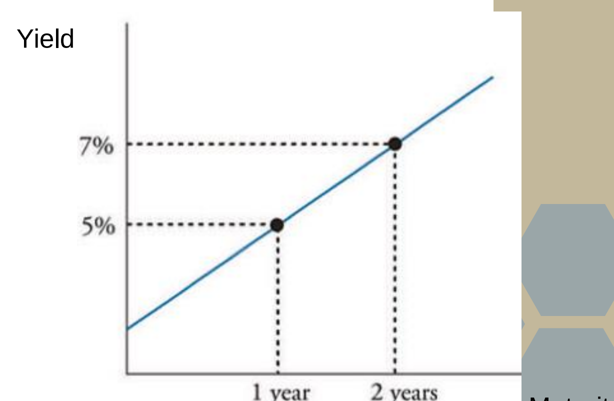
$$= \frac{(1+7\%)^2}{(1+5\%)} - 1$$

$$= 9\%$$

Implied forward rate/Breakeven rate/Lock-in rate/Expected spot rate

➢*Implication for Yield Curve Shape*

Short-term Rates	Shape of Yield Curve
Expected to *rise* in the future	Upward sloping
Expected to *fall* in the future	Downward sloping
Expected to *rise then fall*	humped
Expected to *remain constant*	flat



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Unbiased Expectation Theory

➢ ***Fails to recognize***

- ***Price Risk-*** the uncertainty associated with the future price of a bond that may be sold prior to its maturity.

- ***Reinvestment-risk-*** the uncertainty associated with the rate at which bond cash flows can be reinvested over an investment horizon.

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Traditional Theories of the Term Structure of Interest Rates

Local Expectations Theory

- Similar to the unbiased expectations theory with one major difference:

- Bond *maturity does not influence* returns for short holding periods. This implies that over short time periods, every bond (even long-maturity risky bonds) should earn the risk-free rate.

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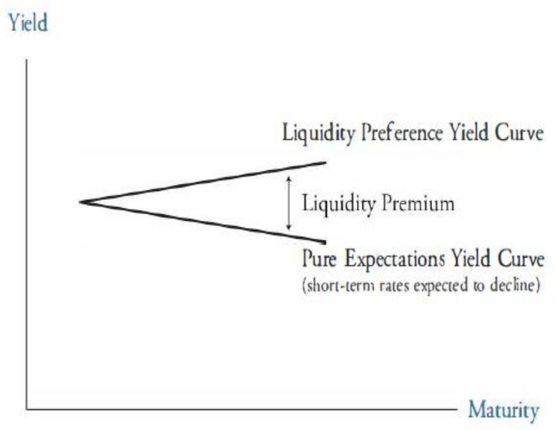
Liquidity Preference Theory

➢*Interpretation of Forward Rates*

- Forward Rates = Expectations of future spot rates + Liquidity premium
- Investors require a liquidity risk premium for holding longer term bonds.
- Liquidity premium is *positively related* to maturity:
  A 25-year bond has a larger liquidity premium than a 5-year bond.

➢*Implication for Yield Curve Shape*

Short-term Rates	Shape of Yield Curve
Expected to rise in the future	Upward sloping
Expected to rise then fall	humped
Expected to fall in the future	Downward or upward sloping



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Segmented Markets Theory

- The shape of the yield curve is determined by the preferences of borrowers and lenders, which drives the balance between supply of and demand for loans of different maturities
- We can think of each maturity to be essentially unrelated to other maturities.

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Preferred Habitat Theory

➢*Interpretation of Forward Rates*

- Forward Rates = Expectations of future spot rates + Risk premium
- Investors require a risk premium to compensate them to move to a *"less-than-preferred"* habitat.
- Premiums are *related to supply and demand* for funds at various maturities:
  A 10-year bond might have a higher or lower risk premium than a 25-year bond.

➢*Implication for Yield Curve Shape*

This theory is consistent with ***any yield curve shape***. It is supply and demand for debt securities at each maturity range that determines the yield for that maturity range.

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Modern Term Structure Models

- Modern Term Structure Models attempt to capture the statistical properties of interest rate movements and provide quantitatively precise descriptions of how interest rates evolve.

Equilibrium Term Structure Models

The Cox-Ingersoll-Ross Model	The Vasicek Model
▪ Both seek to describe the dynamics of the term structure using fundamental economic variables that are assumed to affect interest rates. ▪ Both assume a single factor, the short-term interest rate, r. ▪ Both assume the behavior of the factor (mean reverting). ▪ Both include a drift part and a random part	
$dr = a(b - r)dt + \sigma\sqrt{r}dz$ where: dr = change in the short-term interest rate a = speed of mean reversion parameter (a high *a* means fast mean reversion) b = long-run value of the short-term interest rate r = the short-term interest rate t = time dt = a small increase in time σ = volatility dz = a small random walk movement	$dr = a(b - r)dt + \sigma dz$
▪ The a(b-r)dt term forces the interest rate to mean-revert toward the long-run value b, at a speed determined by the parameter a.	▪ Interest rates are mean reverting to some long-run value.
▪ The random component indicates volatility increases with interest rate.	▪ Volatility does not increase as the level of interest rates increase.
▪ Non-positive interest rates	▪ The main disadvantage of model: it does not force interest rates to be non-negative.

Modern Term Structure Models

Arbitrage-Free Models

- The analysis begins with the observed market prices (assumed to be correctly priced)of a reference set of financial instruments.
- The model generates a term structure that will produces valuation which matches the market prices of the reference set of financial instruments-arbitrage free.

The Ho-Lee Model $dr_t = \theta_t dt + \sigma dz_t$

How the Ho-Lee Model generates the term structure:

EX: Assume that the current short-term rate is 4%. The time step is monthly, and the drift terms, which are determined using market prices ,are θ_1=1% in the first month and θ_2=0.80% in the second month. The annual volatility is 2%. Create a two-period binomial lattice-based model for the short-term rate.

Annual volatility =2% => σ=Monthly Volatility = $\sigma\sqrt{\frac{1}{t}} = 2\%\sqrt{\frac{1}{12}} = 0.5774\%$

dt=a month = 1/12=0.0833

$dr_t = \theta_t\, dt + \sigma\, dz_t = \theta_t\,(0.0833) + (0.5774)dz_t$

If the rate goes up in the 1st month,

$$r = 4\% + (1\%)(0.0833) + 0.5774\% = 4.6607\%$$

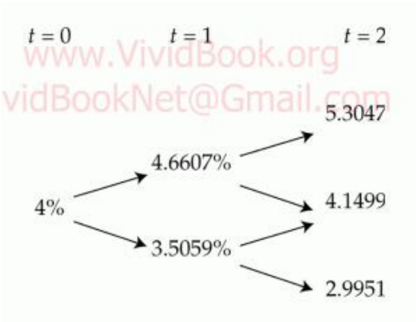
If the rate goes down in the 1st month,

$$r = 4\% + (1\%)(0.0833) - 0.5774\% = 3.5059\%$$

If the rate goes up in the 1st month and up in the 2nd month, r=5.3047%

If the rate goes up(down) in the 1st month and down(up) in the 2nd month, r=4.1499%

If the rate goes down in the 1st month and down in the 2nd month, r=2.9951%



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- The model can be used to price zero-coupon bonds and to determine the spot curve.
- By construction, the model output is consistent with market prices.
- Future rates follow normal distribution, so negative interest rates are possible.

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Framework

Reading

Reading: The Term Structure and Interest Rate Dynamics

- Spot Rates, Forward Rates, Yield Curve
  - Relationships among spot rates, forward rates, YTM and expected and realized return on bonds
  - Forward pricing and forward rate models
  - Evolution of spot rates in relation to forward rates
  - Strategy of riding the yield curve
  - Swap rate curve
- Swap Spread, I-Spread, Z-Spread, TED Spread, LIBOR-OIS Spread
- Traditional Theories of Term Structure of Interest Rates
- Modern Term Structure Models
- Measuring Yield Curve Risk
- Term Structure of Interest Rate Volatility

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Measuring Yield Curve Risk

- **Yield curve risk** refers to risk to the value of a bond portfolio due to unexpected changes in the yield curve.

Effective Duration

- **Effective duration** measures price sensitivity to small parallel shifts in the yield curve.

Key Rate Duration

- **Key rate duration** can be used to measure the value change of a bond portfolio to a *nonparallel shift* in the yield curve.
- **Key rate duration**: Holding all other spot rates constant, given a change in a *single spot rate*, the value change in a security or a portfolio. EX: 3-year key rate duration= 0.2

EX: A bond portfolio has interest rate risk exposure to only 3 maturity points on the spot rate curve, 1, 5, and 25 year maturities, with key rate durations represented by D_1=0.7, D_5=3.5, and D_{25}=9.5 respectively.

- **The Model for yield curve risk** using these key rate durations:

$$\frac{\Delta P}{P} \approx -D_1\Delta r_1 - D_5\Delta r_5 - D_{25}\Delta r_{25} \quad \Rightarrow \quad \frac{\Delta P}{P} \approx -(0.7)\Delta r_1 - (3.5)\Delta r_5 - (9.5)\Delta r_{25}$$

Sensitivity to Parallel, Steepness, and Curvature Movements

- **The model for yield curve risk** using a combination of change in level, steepness and curvature:

$$\frac{\Delta P}{P} \approx -D_L\Delta x_L - D_S\Delta x_S - D_C\Delta x_C$$

where

D_L, D_S, and D_C are respectively the portfolio's sensitivities to changes in the yield curve's level, steepness, and curvature.

Level (Δx_L) – A parallel increase or decrease of interest rates.

Steepness (Δx_S) – Long-term interest rates increase while short-term rates decrease.

Curvature (Δx_C) – Increasing curvature means short- and long-term interest rates increase while intermediate rates do not change.

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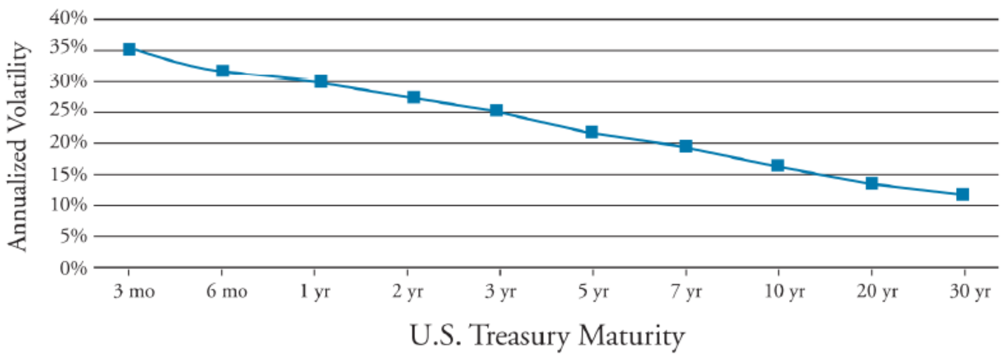
Term Structure of Interest Rate Volatility

- **Term Structure of Interest Rate Volatility**: the graph of yield volatility versus maturity.

Figure 4: Historical Volatility Term Structure: U.S. Treasuries, August 2005–December 2007



- Short-term interest rates are generally **more volatile** than the long-term rates.
  - ✓ Volatility at the long-maturity end: associated with uncertainty regarding the real economy and inflation.
  - ✓ Volatility at the short-maturity end: reflects risks regarding monetary policy.

- $\sigma(t,T)$. : interest rate volatility at time t for a security with maturity of T, measures the annualized standard deviation of the change in bond yield.

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Item Set Example

Eastwood Scenario

(This case was adapted from the CFA mock exams)

Joshua White, CFA, and Stephen Butler are co-managers of Eastwood Core Bond Fund. Eastwood is a fixed income fund that is benchmarked against the U.S. Barclays Aggregate Bond Index. The fund and index contain securities in the Treasury, credit, asset-backed, and mortgage-backed sectors of the market.

White and Butler first discuss their expectations on the direction of interest rates. White states:" Rates are attractive across the curve. The 7- to 10-year part of the curve looks expensive, but that should not deter us because it is driven by insurance companies hedging their liabilities." Butler responds:" Interest rates for long maturity bonds look attractive; the risk premium appears to compensate us for the potential downside of adding duration. This premium is above the expected forward rates."

White then discusses yield spread with Deb Miller, CFA, Eastwood's corporate bond analyst. Miller offers the following observations:

Observation1:If the three-month T-bill rate drops and the Libor rate remains the same, the relevant TED spread increases.

Observation2:I-spread not only reflects time value, but also compensation for credit and liquidity risk.

Observation3: Given the yield curve for US Treasury zero-coupon bonds, Z-spread is more helpful pricing a corporate bond than TED spread and Libor-OIS spread.

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Item Set Example

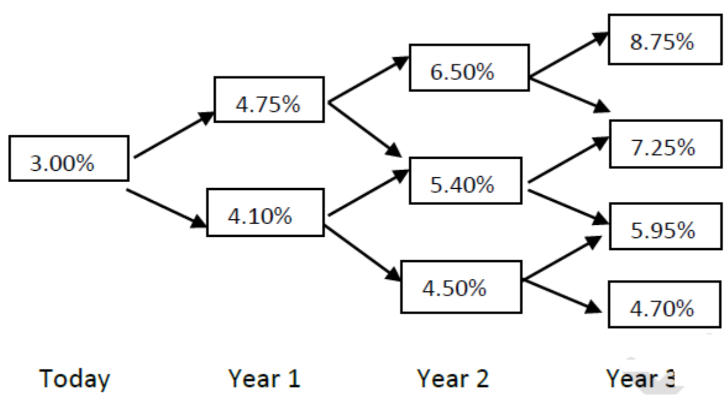
Butler then focuses on the mortgage securities in the portfolio. He asks Miller to explain what the cash flow implications are for a pool of mortgages in the portfolio. Butler describes the mortgages in the pool as having a 20-month average age, and the pool has a monthly mortality of 0.4353%.

Miller then offers to go over her valuation of a callable bond issue by a company she has been researching. The bond is callable at $101.50 every year starting one year from today. She uses the data in Exhibit 1 for her valuation.

Exhibit 1

Binomial Interest Rate Tree (9% Volatility Assumed)

for Valuing a $100 Par Value 3-Year Callable Bond with a 6.25% Coupon



Butler tests Miller's knowledge of securitized transactions by asking her to explain the tranches of the ABS securitization in Exhibit 2.

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Item Set Example

Exhibit 2 ABS Structure

Bond Class	Par Value ($millions)
A1 (senior)	40
A2 (senior)	25
A3 (senior)	20
B (subordinate)	8
C (subordinate)	7
Total	**100**

Miller provides the following explanations:" This securitization is a sequential-pay transaction. As such, interest payments are paid to each bond class periodically. Principal repayments are applied first to the lowest tranche, in this case tranche C, to protect investors from prepayment risk. The senior-subordinate structure has been established for credit tranching to protect against defaults, with subordinated tranches sharing equally in any losses".

White then asks Miller which of the various valuation models would be most appropriate for assessing relative value. Costas responds:" It really depends on the characteristics of the security. As examples, consider the following three securities."

Security A: 5%, non-callable 30-year corporate bond selling at a discount.
Security B: 4%, 20-year Ginnie Mae debenture callable in five years.
Security C: Zero-coupon, 10-year Treasury bond.

Miller explains that the most appropriate measure to use are a zero-volatility spread for Security A, an option-adjusted spread (OAD) for Security B, and a nominal spread for Security C.

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Item Set Example

1. Which theory of the term structure of interest rates least likely explains the views of either White or Butler
A. Preferred habitat
B. Pure expectations
C. Liquidity preference

Answer:B

2. In observing yield spread, Miller is least likely correct respect to:
A. Observation 1
B. Observation 2
C. Observation 3

Answer:B

3. The prepayment estimate of the mortgage pool Butler describes is closest to a PSA of :
A. 85%
B. 128%
C. 131%

4. Using the data in Exhibit 1, the current value of the callable bond Miller is analyzing is closes to:

A. 105.56
B. 104.61
C. 101.40

5. Miller's explanation of the securitization in Exhibit 2 is least likely correct with respect to:
A. Interest payments and losses
B. Losses and principal payments
C. Interest and principal payments

6. Miller is least likely correct with respect to the valuation measure for:

A. Security A
B. Security B
C. Security C

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Thank You!