CFAspace

Provided by APF

Academy of Professional Finance 专业金融学院

CFA Level II

Portfolio Management:

An Introduction to Multifactor Models

Lecturer: Nan Chen

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes
Reading: Analysis of Active Portfolio Management	Yes
Reading: Economics and Investment Markets	Yes
Reading: The Portfolio Management Process	No

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Item Set Example

Joshua White Scenario

(This case was adapted from the CFA mock exams)

Joshua White is an equity portfolio manager for Eastwood Investments, who currently uses the Capital Asset Pricing Model (CAPM) to evaluate securities and mean-variance portfolio optimization to construct equity portfolios. White is meeting with two assistant portfolio managers, Stephen Butler and Deb Miller . Butler and Miller have been asked to do some research on ways to improve on the methods currently being used by Eastwood to evaluate securities and develop portfolios.

White begins the meeting by outlining some issues relating to the CAPM. He makes the following statements:

Statement 1

"One of the reasons I am uncomfortable using the CAPM is that it makes some very restrictive assumptions such as :

*investors pay no taxes on returns and no transaction costs on trades,

*investors have unique views on expected returns, variances and correlations of securities, and

*investors can borrow and lend at the same risk-free rate of interest."

Statement 2

"We are also faced with a problem that our mean-variance optimization models can generate unstable minimum-variance efficient frontiers. Consequently, we face considerable uncertainty regarding recommendations we make to our clients on asset allocation. I attribute the instability to our use of:

A short sales constraint, and

Historical betas."

Item Set Example

Butler suggests that multifactor models provide a better way to model stock returns. He states that "there are two ways to model stock returns using the following multifactor model:
$R_i=a_i+b_{i1}F_1+b_{i2}F_2+\ldots+b_{ik}F_k+\varepsilon_i$

Model1: In this model, stock returns (R_i) are determined by surprises in economic factors such as GDP growth and the level of interest rates.

Model2: Here, stock returns (R_i) are determined by factors that are company attributes such as PE ratios and market capitalization.

While the interpretation of the intercept a_i is similar for both models, the factor sensitivities b_i are interpreted differently in the two models."

Miller notes that a multifactor Arbitrage Pricing Model (APT) provides a much better basis than the CAPM for calculating expected portfolio returns and evaluating portfolio risk exposures. In order to illustrate the advantages of the multifactor APT model, Miller provides information for two portfolios Eastwood currently manages. The information is provided below in the exhibit. The current risk-free rate is 2%.

Factor Sensitivities and Risk Premia

Risk Factor	Factor Sensitivities			Factor Risk Premium (%)
	Portfolio A	Portfolio B	Benchmark	
Confidence Risk	0.81	0.04	0.5	4.5
Inflation Risk	-0.15	-0.45	-0.25	-1.2
Business Cycle Risk	1.23	0.09	0.9	5.2

Item Set Example

Miller makes the following statement:

Statement 3
"We can tell from the exhibit that Portfolio A is structured in such a manner that it will benefit from an expanding economy and improving confidence because the factor sensitivities for confidence risk and business cycle risk exceed the factor sensitivities for the benchmark. Portfolio B has very low factor sensitivities for confidence risk and business cycle risk but moderately high exposure to inflation risk, therefore Portfolio B can be referred to as a factor portfolio for inflation risk."

White wants to examine how active management is contributing to portfolio performance. Miller responds with the following statement:

Statement 4
"Our models show that Portfolio A has annual tracking error of 1.25% and an information ratio of 1.2 while Portfolio B has an annual tracking error of 0.75% and an information ratio of 0.87."

Item Set Example

1. Which assumption of the CAPM is *most likely* incorrect in White's statement 1? The assumption regarding:
A. Borrowing and lending
B. Taxes and transaction costs
C. Expected returns, variances and correlations

2. Is White's statement 2 most likely correct?
A. Yes.
B. No, she is incorrect about the short sales constraint.
C. No, she is incorrect about the use of historical betas.

3. With regard to the statement on multifactor models, Butler is most likely incorrect with respect to the:
A. Intercept value a_i.
B. Factor sensitivities b_i.
C. Description of the factors.

4. Based on the information in the exhibit, the expected return for portfolio A is closest to:
A. 8.4%
B. 10.2%
C. 12.2%

5. Is Miller's Statement 3 most likely correct?
A. Yes.
B. No, she is incorrect about Portfolio A.
C. No, she is incorrect about Portfolio B.

6. Based on Statement 4 by Miller, an appropriate conclusion is that the portfolio that has benefited the most from active management is:
A. Portfolio B because of tracking error.
B. Portfolio A because of the information ratio.
C. Portfolio B because of the information ratio.

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Arbitrage Pricing Theory (APT)

➢ Three ***Assumptions*** of Arbitrage Pricing Theory (APT):

- ✓ Unsystematic risk can be completely diversified away.
- ✓ Returns are derived from a multifactor model.
- ✓ No arbitrage opportunities exist.
  - *An arbitrage opportunity* is defined as an investment opportunity that bears no risk, no cost, and yet provides a profit.

➢ **Arbitrage Pricing Model**:

- *Equilibrium* relationship between *expected returns* for well diversified portfolios and their multiple sources of *systematic risk*.

$$E(R_P) = R_F + \beta_{P,1}(\lambda_1) + \beta_{P,2}(\lambda_2) + \ldots + \beta_{P,k}(\lambda_k)$$

- $E(R_P)$: Expected return on Portfolio P
- $\beta_{P,1}(\lambda_1)$: Risk premium associated with each risk factor
- $\beta_{P,k}(\lambda_k)$: Sensitivity of Portfolio P to each risk factor

➢ **Compare** CAPM and APT Model:

- ✓ The CAPM can be considered *a special restrictive case* of the APT in which there is only one risk factor (market risk factor).
- ✓ The APT does NOT require that one of the risk factors is the market risk factor, which is a *major advantage* of the arbitrage pricing model.

Arbitrage Pricing Theory (APT)

➢ **Calculate** expected returns from the Arbitrage Pricing Model:

EX: A two-factor APT model was adopted by Eastwood Investment Firm. Calculate the expected return for one of the firm's portfolios using the following data:

The risk-free rate equals 5%.

	Factor 1	Factor 2
Factor Risk Premiums	1.5	2.0
Factor Sensitivities	0.0300	0.0125

➢the *expected return on the Portfolio*:

$$E(R_P) = R_F + \beta_{P,1}(\lambda_1) + \beta_{P,2}(\lambda_2) + \ldots + \beta_{P,k}(\lambda_k)$$

$E(R_p) = 0.05 + 1.5 \times 0.0300 + 2.0 \times 0.0125 = 12\%$

Arbitrage Pricing Theory (APT)

➢ **How to exploit** an arbitrage opportunity:

EX: Determine whether an arbitrage opportunity exists from the data below: Assume Eastwood Investment Firm uses a single factor model to evaluate assets. Information related to portfolios A, B, and C are provided:

Portfolio	Expected Return	Beta
A	10%	1.0
B	20%	2.0
C	13%	1.5

➢By allocating 50/50 between portfolios A and B, we can obtain a portfolio D with the same beta as that of portfolio C, because 0.5(1)+0.5(2) = 1.5

=>Now portfolio D and portfolio C has the same beta, i.e. the *same risk*.

➢E(RD)=0.5(0.1)+0.5(0.2) = 15%

=> Despite the same risk, portfolio D has *higher expected return* than portfolio C; portfolio D is undervalued.

➢By purchasing portfolio D and short-selling portfolio C , an arbitrage profit could be exploited.

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Multifactor Models

Macroeconomic Factor Models

Fundamental Factor Models

Statistical Factor Models:

- ✓ Use statistical methods to explain asset returns.
- ✓ Major weakness: the statistical factors do not lend themselves well to economic interpretation.

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Macroeconomic Factor Models

- A two-factor macroeconomic model:

$$R_i = E(R_i) + b_{i1}F_{GDP} + b_{i2}F_{QS} + \varepsilon_i$$

- Describe the two-factor macroeconomic model:

Systematic Risk Factors (Priced)	F_{GDP}: surprises in the GDP rate	"F's" (factor surprises): the differences between the *predicted* value of the factor and the *realized* value.
	F_{QS}: surprise in the credit quality spread	
Unsystematic Risk : can be diversified away (NOT priced)	ε_i : the part of the return that CANNOT be explained by the model.	
Factor Sensitivities	b_{i1} for retail stocks: larger	"b's": sensitivities of the stock to the surprises;
	b_{i1} for food stocks : smaller	
Expected return	$E(R_i)$: intercept	

- We should be able to compute a stock return using a macroeconomic factor model.

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Fundamental Factor Models

- An example of fundamental factor model:

$$R_i = a_i + b_{i1}F_{P/E} + b_{i2}F_{SIZE} + \varepsilon_i$$

Describe the fundamental factor model:

$F_{P/E}$: rate of return associated with the PE factor

F_{SIZE}: rate of return associated with the size factor

$$b_{i1} = \frac{(P/E)_i - \overline{P/E}}{\sigma_{P/E}}$$

If b_{i1}=2, Stock i has a P/E that is 2 standard deviations above the mean.

Factor sensitivities:
For *macroeconomic* factor models: regression slopes;
For *fundamental* factor models: standardized attributes.

Intercept a_i: NOT interpreted as expected return

EX: The P/E ratio for Stock HT is 15.20, the average P/E ratio for all stocks is 11.90, and the standard deviation of P/E ratios is 6.30. Calculate the standardized sensitivity of Stock HT to the P/E factor.

- Standardized Sensitivity of Stock HT to the P/E factor

  =(15.20-11.90)/6.30

  =0.52

- Interpretation: the P/E ratio for the stock is 0.52 standard deviation above the average stock P/E.

Fundamental Factor Models

Compare

	Macroeconomic Factor Model	Fundamental Factor Model
Sensitivities	Slope *estimates* from regression	*Calculated* from the attribute data (ex: P/E); not estimated
Interpretation of factors	Factors are *surprises* in the macroeconomic variables	Factors are *rates of return* associated with each factor
Number of factors	Small in number	Large in number
Intercept term	Equals the stock's expected return	Does not equal the stock's expected return; Has no economic interpretation.

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Application of Multifactor Models

- **Active Return** = $R_P - R_B$
- **Active Risk** / Tracking Error / Tracking Risk = s ($R_P - R_B$)
- **Information Ratio** = $\frac{\bar{R}_P - \bar{R}_B}{s_{(R_P - R_B)}}$ *demonstrates a manager's **consistency** in generating active return*

	Q1	Q2	Q3	Q4	Average Active Return	Active Risk	Information Ratio
Manager A	0.6%	0.5%	0.5%	0.4%	0.5%	0.000816	6.12
Manager B	8%	5%	-3%	-8%	0.5%	0.073258	0.07

- **Information Ratio vs. Sharpe Ratio**

$$IR = \frac{\overline{R_P} - \overline{R_B}}{s_{(R_P - R_B)}} \quad \textit{vs.} \quad SR = \frac{R_P - R_F}{\sigma_P}$$

Application of Multifactor Models – Return Attribution

- **Active Return** = $R_P - R_B$
- **Active Return**= Factor Return + Security Selection Return
  - ✓ *Factor Return*: arising from the manager's decision to take on factor exposures that differ from those of the benchmark.

$$\text{factor return} = \sum_{i=1}^{k} (\beta_{pk} - \beta_{bk}) \times (\lambda_k)$$

β_{pk} = factor sensitivity for the kth factor in the active portfolio
β_{bk} = factor sensitivity for the kth factor in the benchmark portfolio
λ_k = factor risk premium for factor k

  - ✓ *Security Selection Return*: arising from the manager choosing a different weight for specific securities compared to the weight of those securities in the benchmark.

security selection return = active return – factor return

Application of Multifactor Models – Risk Attribution

- **Active Risk** / Tracking Error / Tracking Risk = s (R_P –R_B)
- **Active Risk Squared** = Active Factor Risk + Active Specific Risk
  - ✓ *Active Factor Risk*: risk from active factor tilts attributable to under or overweighting particular industries relative to the portfolio's benchmark.

active factor risk = active risk squared – active specific risk

  - ✓ *Active Specific Risk*: risk from active asset selection attributable to deviations of the portfolio's individual asset weightings versus the benchmark's individual asset weightings.

$$\text{active specific risk} = \sum_{i=1}^{n}(W_{pi} - W_{bi})^2 \sigma_{\varepsilon i}^{\ 2}$$

where:

W_{pi} and W_{bi} = weight of ith security in the active and benchmark portfolio, respectively

$\sigma_{\varepsilon i}^{\ 2}$ = residual (i.e., unsystematic) risk of the ith asset

Application of Multifactor Models – Risk Attribution

EX: A fund analyst is analyzing the performance of three actively managed mutual funds using a two-factor model. The results of risk decomposition are shown below:

Fund	Active Factor			Active Specific	Active Risk Squared
	Size Factor	Style Factor	Total Factor		
A	6.25	12.22	18.47	3.22	21.69
B	3.20	0.80	4.00	12.22	16.22
C	17.85	0.11	17.96	19.7	37.66

Which fund assumes the highest level of active risk, the highest level of style factor risk as a proportion of active risk, the highest level of size factor risk as a proportion of active risk, and the lowest level of active specific risk as a proportion of active risk?

➢Proportional contributions of various sources of active risk as a proportion of active risk squared:

Fund	Active Factor			Active Specific	Active Risk
	Size Factor	Style Factor	Total Factor		
A	29%	56%	85%	15%	4.7%
B	20%	5%	25%	75%	4.0%
C	47%	0%	48%	52%	6.1%

➢**Fund C** assumes the highest level of active risk; **Fund A** assumes the highest level of style factor risk; **Fund C** assumes the highest level of size factor risk; **Fund A** assumes the lowest level of active specific risk.

Application of Multifactor Models – Portfolio Construction

- **Passive Management:**
  Managers seeking to track a benchmark can construct a tracking portfolio.

  - ***Tracking Portfolio*** is a portfolio having factor sensitivities that are matched to those of a benchmark or other portfolio.

Application of Multifactor Models – Portfolio Construction

➢ Active Management

✓ ***Factor Portfolio*** for a particular factor has a sensitivity of 1 for that factor and a sensitivity of 0 for all other factors.

- *Factor portfolios can be used to **hedge** that factor risk (offset it) or **speculate** on it.*

Risk Factor	Factor Sensitivities
Confidence Risk	*0*
Time Horizon Risk	*0*
Inflation Risk	*0*
Business Cycle Risk	*1*
Market Timing Risk	*0*

- *This portfolio is a factor portfolio for business cycle risk.*
- ***Speculate**: The portfolio manager would take a long position in the factor portfolio to place a bet on an increase in real business activity.*
- ***Hedge**: The portfolio manage would take a short position in the factor portfolio to hedge an existing positive exposure to business cycle risk.*

$$E(R) = R_F + \beta_1 RMRF + \beta_2 SMB + \beta_3 HML + \beta_4 WML$$

EX: The table below provides the factor exposures of three portfolios based on Carhart Model:

Portfolio	Risk Factor			
	RMRF	SMB	HML	WML
A	1.00	0.00	0.00	0.00
B	0.00	1.00	0.00	0.00
C	1.20	0.00	0.20	0.80

Which strategy would be most appropriate if the manager expects that:

A. RMRF will be higher than expected.

B. Large cap stocks will outperform small cap stocks.

➢A. go **long** on Portfolio A, which is constructed to be a pure bet on RMRF factor.

➢B. go **short** on Portfolio B, which is constructed to be a pure bet on SMB factor.

Application of Multifactor Models – Portfolio Construction

➢ Rules-based active management (alternative indices):

- ✓ These strategies routinely tilt toward factors such as size, value, quality, or momentum when constructing portfolios.
- ✓ These strategies introduce biases in the portfolio relative to market capitalization-weighted indexes.

Framework

Reading	Changes
Reading: An Introduction to Multifactor Models	Yes

- Arbitrage Pricing Theory (APT)
- Multifactor Models
  - Macroeconomic Factor Models
  - Fundamental Factor Models
  - Application of Multifactor Models
  - Benefits in Considering Multiple Risk Factors

Benefits in Considering Multiple Risk Factors

- By including more risk factors, multifactor models enable investors to
  - ✓ zero in on risks that the investor has a comparative advantage in bearing and
  - ✓ Avoid the risks that the investor is incapable of absorbing.

- University endowments
  - ✓ Comparative advantage – business cycle risk and liquidity risk
  - ✓ Comparative disadvantage – inflation risk

- CAPM Framework vs. Multifactor Model:
  - ✓ Under the CAPM framework, investors choose a combination of the market portfolio and the risk-free asset.
  - ✓ A multifactor approach offer a richer combination of SMB, HML, and WML factors in addition to the market factor, and thus can help investors achieve better-diversified and possibly more efficient portfolios.

Item Set Example

Joshua White Scenario

(This case was adapted from the CFA mock exams)

Joshua White is an equity portfolio manager for Eastwood Investments, who currently uses the Capital Asset Pricing Model (CAPM) to evaluate securities and mean-variance portfolio optimization to construct equity portfolios. White is meeting to two assistant portfolio managers, Stephen Butler and Deb Miller . Butler and Miller have been asked to do some research on ways to improve on the methods currently being used by Eastwood to evaluate securities and develop portfolios.

White begins the meeting by outlining some issues relating to the CAPM. He makes the following statements:

Statement 1

"One of the reasons I am uncomfortable using the CAPM is that it makes some very restrictive assumptions such as :

*investors pay no taxes on returns and no transaction costs on trades,

*investors have unique views on expected returns, variances and correlations of securities, and

*investors can borrow and lend at the same risk-free rate of interest."

Statement 2

"We are also faced with a problem that our mean-variance optimization models can generate unstable minimum-variance efficient frontiers. Consequently, we face considerable uncertainty regarding recommendations we make to our clients on asset allocation. I attribute the instability to our use of:

- A short sales constraint, and
- Historical betas."

Item Set Example

Butler suggests that multifactor models provide a better way to model stock returns. He states that "there are two ways to model stock returns using the following multifactor model:
$R_i=a_i+b_{i1}F_1+b_{i2}F_2+\ldots+b_{ik}F_k+\varepsilon_i$

Model1: In this model, stock returns (R_i) are determined by surprises in economic factors such as GDP growth and the level of interest rates.

Model2: Here, stock returns (R_i) are determined by factors that are company attributes such as PE ratios and market capitalization.

While the interpretation of the intercept a_i is similar for both models, the factor sensitivities b_i are interpreted differently in the two models."

Miller notes that a multifactor Arbitrage Pricing Model (APT) provides a much better basis than the CAPM for calculating expected portfolio returns and evaluating portfolio risk exposures. In order to illustrate the advantages of the multifactor APT model, Miller provides information for two portfolios Eastwood currently manages. The information is provided below in the exhibit. The current risk-free rate is 2%. $E(R_p) = R_F + \beta_{P,1}(\lambda_1) + \beta_{P,2}(\lambda_2) + \ldots + \beta_{P,k}(\lambda_k)$ =2%+0.81*4.5%-0.15*(-1.2%)+1.23*5.2%=12.2%

Factor Sensitivities and Risk Premia

Risk Factor	Factor Sensitivities			Factor Risk Premium (%)
	Portfolio A	Portfolio B	Benchmark	
Confidence Risk	0.81	0.04	0.5	4.5
Inflation Risk	-0.15	-0.45	-0.25	-1.2
Business Cycle Risk	1.23	0.09	0.9	5.2

Item Set Example

Miller makes the following statement:

Statement 3
"We can tell from the exhibit that Portfolio A is structured in such a manner that it will benefit from an expanding economy and improving confidence because the factor sensitivities for confidence risk and business cycle risk exceed the factor sensitivities for the benchmark. Portfolio B has very low factor sensitivities for confidence risk and business cycle risk but moderately high exposure to inflation risk, therefore Portfolio B can be referred to as a factor portfolio for inflation risk."

White wants to examine how active management is contributing to portfolio performance. Miller responds with the following statement:

Statement 4
"Our models show that Portfolio A has annual tracking error of 1.25% and an information ratio of 1.2 while Portfolio B has an annual tracking error of 0.75% and an information ratio of 0.87."

Factor Sensitivities and Risk Premia

Risk Factor	Factor Sensitivities			Factor Risk Premium (%)
	Portfolio A	Portfolio B	Benchmark	
Confidence Risk	0.81	0.04	0.5	4.5
Inflation Risk	-0.15	-0.45	-0.25	-1.2
Business Cycle Risk	1.23	0.09	0.9	5.2

Item Set Example

1. Which assumption of the CAPM is *most likely* incorrect in White's statement 1? The assumption regarding:
A. Borrowing and lending
B. Taxes and transaction costs
C. Expected returns, variances and correlations

2. Is White's statement 2 most likely correct?
A. Yes.
B. No, she is incorrect about the short sales constraint.
C. No, she is incorrect about the use of historical betas.

3. With regard to the statement on multifactor models, Butler is most likely incorrect with respect to the:
A. Intercept value a_i.
B. Factor sensitivities b_i.
C. Description of the factors.

Answer:A

4. Based on the information in the exhibit, the expected return for portfolio A is closest to:
A. 8.4%
B. 10.2%
C. 12.2%

Answer:C

5. Is Miller's Statement 3 most likely correct?
A. Yes.
B. No, she is incorrect about Portfolio A.
C. No, she is incorrect about Portfolio B.

Answer:C

6. Based on Statement 4 by Miller, an appropriate conclusion is that the portfolio that has benefited the most from active management is:
A. Portfolio B because of tracking error.
B. Portfolio A because of the information ratio.
C. Portfolio B because of the information ratio.

Answer:B

Thank You!