CFAspace

CFA Level I

The Time Value of Money

Part I

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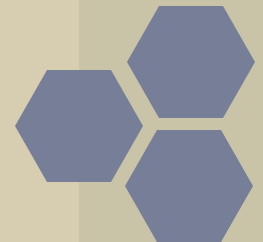
Content

Interest Rate (Los a, b)	Required rate of return
	Discount rate
	Opportunity cost
	Real risk free rate plus premium

Effective Annual Rate (Los c, d)	Stated annual interest rate
	Frequency of compounding
	Continuous compounding

Future Value and Present Value (Los e, f)	Ordinary annuity
	Annuity due
	Perpetuity (PV only)
	A series of unequal cash flows
	Compounding Period other than annual

Time Value of Money

Interest rate, i, is the measure of the time value of money (TVM)

Interest Rate
- Required rate of return
- Discount rate
- Opportunity cost
- Cost of capital

i = real risk-free rate + inflation premium + different types of risk premium

nominal risk-free rate

↑

T-bill rates

- Default risk
- Liquidity risk (lower, higher)
- Maturity risk (longer, higher)

Time Value of Money

Effective Annual Rate (Los c)

Financial institutions usually quote rates as stated annual interest rates, along with a compounding frequency.

To get the actual interest earned by the investors, we use EAR.

EAR = $(1 + \text{periodic rate})^m - 1$

m – the number of compounding periods per year

periodic rate – stated annual rate / m

If the stated annual rate is 10%, what is the EAR for semiannual, quarterly, monthly, and daily compounding?
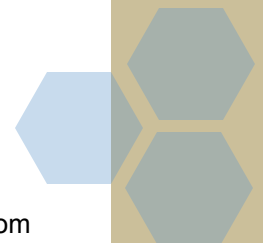
EAR of semiannual compounding = $(1 + 10\% / 2)^2 - 1 = 10.25\%$

EAR of quarterly compounding = $(1 + 10\% / 4)^4 - 1 = 10.38\%$

EAR of monthly compounding = $(1 + 10\% / 12)^{12} - 1 = 10.47\%$

EAR of daily compounding = $(1 + 10\% / 365)^{365} - 1 = 10.515\%$

EAR of continuous compounding = $e^r - 1 = e^{10\%} - 1 = 10.517\%$

Time Value of Money

Effective Annual Rate (Los d)

If the compounding periods are not annual, what should we do?

Mike invests $10,000 in an account that can earn 3% per year with monthly compounding. How much will he get after two years?

EAR = $(1 + 3\% /12)^{12} - 1 = 3.04\%$, $10,000 \times (1 + 3.04\%)^2 = 10,617.57$

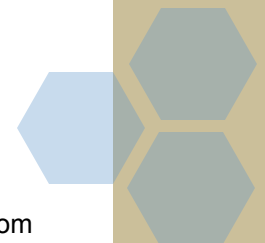There are 24 months and the periodic rate is 3% / 12 = 0.25%, so $10,000 \times (1 + 3\% / 12)^{24} = 10,617.57$

Peter wants to get $10,000 in five years. The return on his account is 5% per year with quarterly compounding. How much should he deposit today in order to meet his goal?

EAR = $(1 + 5\% /4)^4 - 1 = 5.095\%$, $10,000 / (1 + 5.095\%)^5 = 7,800$

There are 20 quarters and the periodic rate is 5% / 4 = 1.25%, so $10,000 / (1 + 5\% / 4)^{20} = 7,800$

Time Value of Money

How to use the calculator

Set P/Y = 1
[2nd][P/Y] "1" [ENTER][2nd][QUIT]

Switch between BGN and END mode
[2nd][BGN][2nd] [SET][2nd][QUIT]
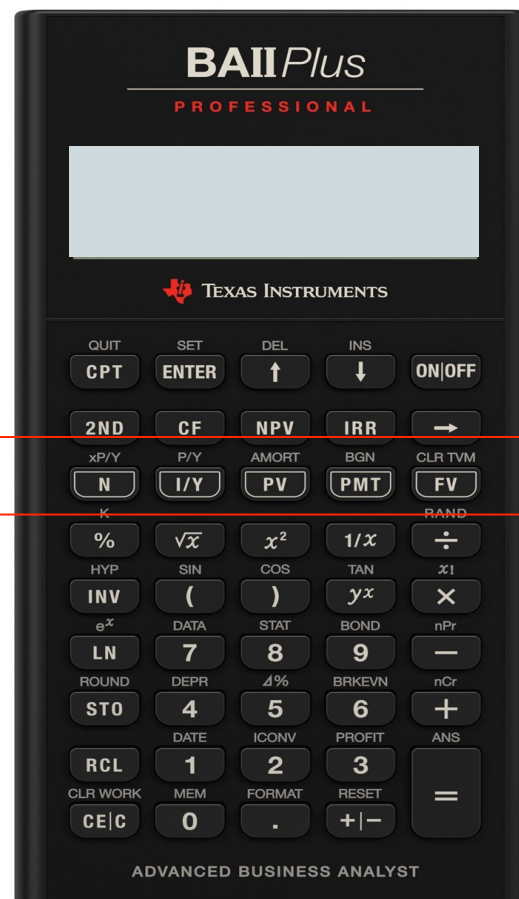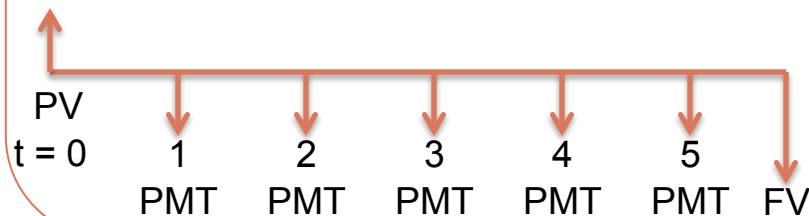
N: Number of compounding periods

I/Y: Interest rate per compounding period

PV: Present Value

FV: Future Value

PMT: Constant periodic cash flow

CPT: Compute

PV
t = 0   1   2   3   4   5   FV
       PMT   PMT   PMT   PMT   PMT

Time Value of Money

1. FV of single sum $\quad$ $FV = PV(1 + I/Y)^N$

Calculate the FV of a \$500 investment at the end of five years if the annually compounded rate of return is 6%.

N = 5; I/Y = 6; PV = -500; PMT = 0; CPT FV = 669.11

2. PV of single sum $\quad$ $PV = FV / (1 + I/Y)^N$

Calculate the PV of a \$2,000 cash flow that will be received in 6 years, given the discount rate of 8%.
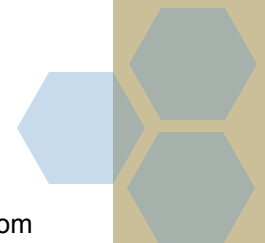
N = 6; I/Y = 8; FV = 2,000; PMT = 0; CPT PV = -1,260.34

The negative sign means the opposite cash flow direction of FV and PV.

3. Annuity

Annuity is a stream of equal cash flows that occurs at equal intervals over a given period.

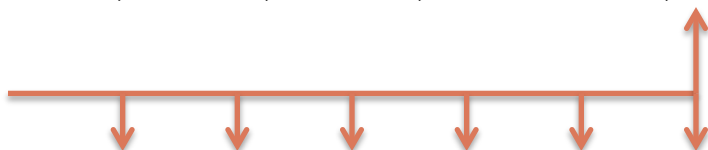Two types of annuity: Ordinary Annuity / Annuity due

Time Value of Money

a) FV of ordinary annuity

Calculate the FV of an ordinary annuity that pays $500 per year for six years, assuming the expected return is 9% each year.
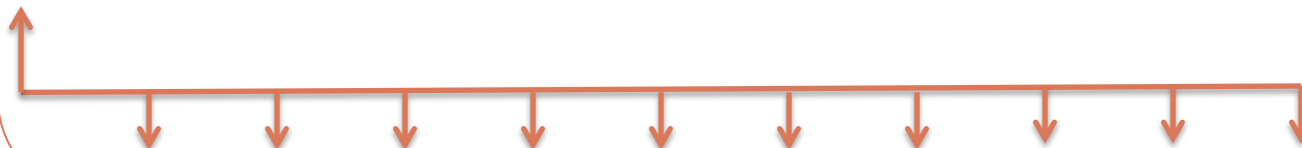
N = 6; I/Y = 9; PV = 0; PMT = -500; CPT FV = 3,761.66

b) PV of ordinary annuity

Calculate the PV of an annuity that pays $2,000 per year for 10 years, given the discount rate of 8%.
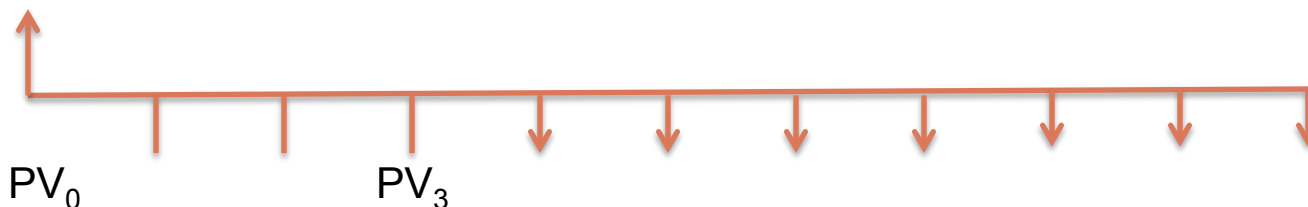
N = 10; I/Y = 8; FV = 0; PMT = -2,000; CPT PV = 13,420.16

Time Value of Money

c) PV of ordinary annuity beginning later than t = 1

Calculate the PV of an annuity that pays $2,000 per year for 7 years and the first payment is to be received four years from today, given the discount rate of 8%.
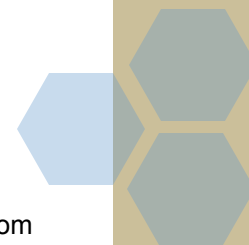
PV_0 PV_3

Step 1

N = 7; I/Y = 8; FV = 0; PMT = -2,000; CPT PV_3 = 10,412.74

Step 2

N = 3; I/Y = 8; FV = -PV_3 = -10,412.74; PMT = 0; CPT PV_0 = 8,265.97
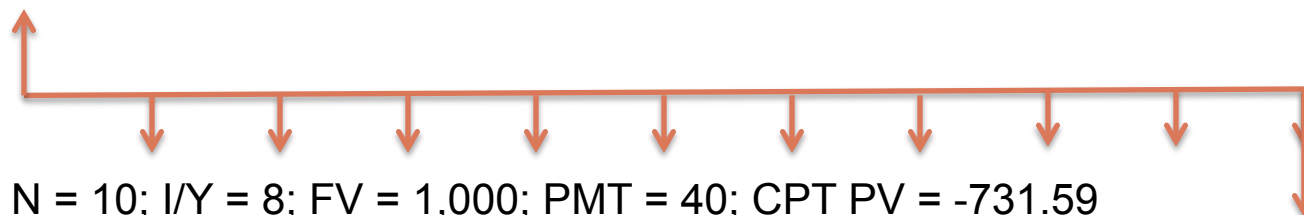
Time Value of Money

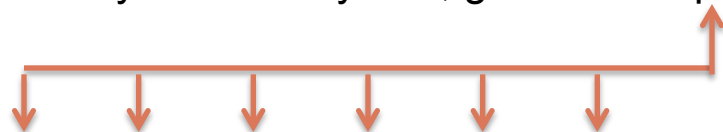**Calculate FV and PV (Los e)**

4. PV of a bond cash flow

Calculate the PV of a bond that pays $40 of interest each year and its par value of $1,000 at maturity in 10 years, given the discount rate of 8%.

N = 10; I/Y = 8; FV = 1,000; PMT = 40; CPT PV = -731.59

5. FV of an annuity due

Calculate the FV of an annuity due that pays $500 at the beginning of each year for six years, given the expected return is 9% each year.

N = 6; I/Y = 9; PV = 0; PMT = -500; CPT FV_5 = 3,761.66;

$FV_6 = FV_5 \times (1 + 9\%) = 4,100$

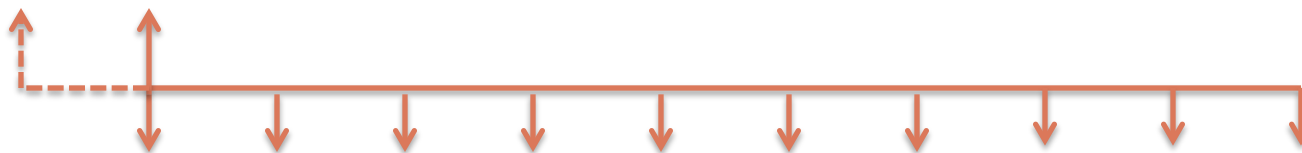## Calculate FV and PV (Los e)

6. PV of an annuity due

Calculate the PV of an annuity that pays $2,000 per year for 10 years, given the discount rate of 8%.

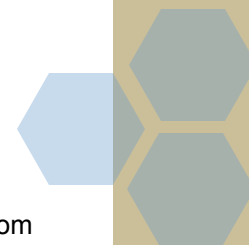N = 10; I/Y = 8; FV = 0; PMT = -2,000; CPT PV_{-1} = 13,420.16

$PV_0 = PV_{-1} \times (1+ 8\%) = 14,493.77$

7. PV of a perpetuity

$PV_{perpetuity} = PMT / (I/Y)$

Calculate the PV of a perpetuity that pays $2,000 per year forever, given the discount rate of 8%.
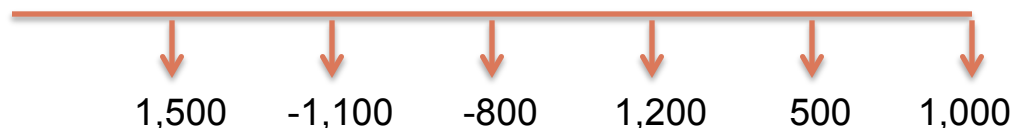
$PV_{perpetuity} = 2,000 / 8\% = 25,000$

8. FV and PV of Uneven Cash Flow Series

1,500	-1,100	-800	1,200	500	1,000

$FV_1 = 1,500 \times (1 + 8\%)^5 = 2,204$

$FV_2 = -1,100 \times (1 + 8\%)^4 = -1,496.54$

$FV_3 = -800 \times (1 + 8\%)^3 = -1,007.77$

$FV_4 = 1,200 \times (1 + 8\%)^2 = 1,399.68$

$FV_5 = 500 \times (1 + 8\%)^1 = 540$

$FV_6 = 1,000 \times (1 + 8\%)^0 = 1,000$

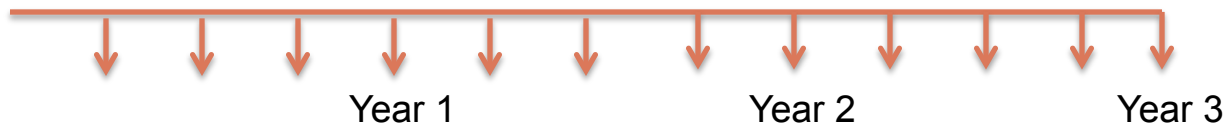$FV = 2,204 + (-1,496.54) + (-1007.77) + 1,399.68 + 540 + 1,000 = 2,639.37$

Time Value of Money

9. Compounding Period other than annual

Calculate the FV of paying $1000 quarterly for three years, given the interest rate of 8%.

N = 3 x 4 = 12; I/Y = 8 / 4 = 2; PMT = -1000; CPT FV = 14,339.93

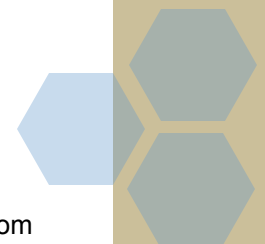Year 1   Year 2   Year 3

10. Loan Payment

Calculate the amount of payment that a company must make quarterly to fully amortize its five-year loan of $60,000 at a rate of 12%.

N = 5 x 4 = 20; I/Y = 12 / 4 = 3; PV = -60,000; FV = 0;

CPT PMT = 4,032.94

Thank You!