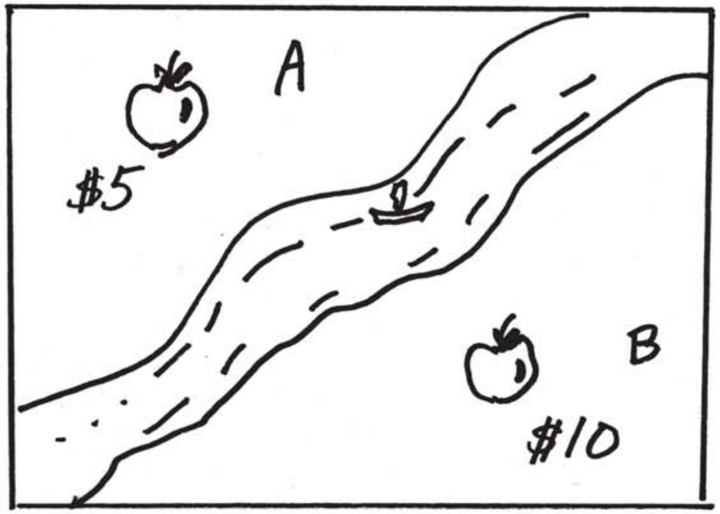
What is Arbitrage ? Why arbitrage process is necessary for price determination?



Arbitrage Condition

Investment	Risk	Return
∅	∅	+
+	∅	Unique Return

Arbitrage = Free Lunch ; when arbitrage profit → 0,

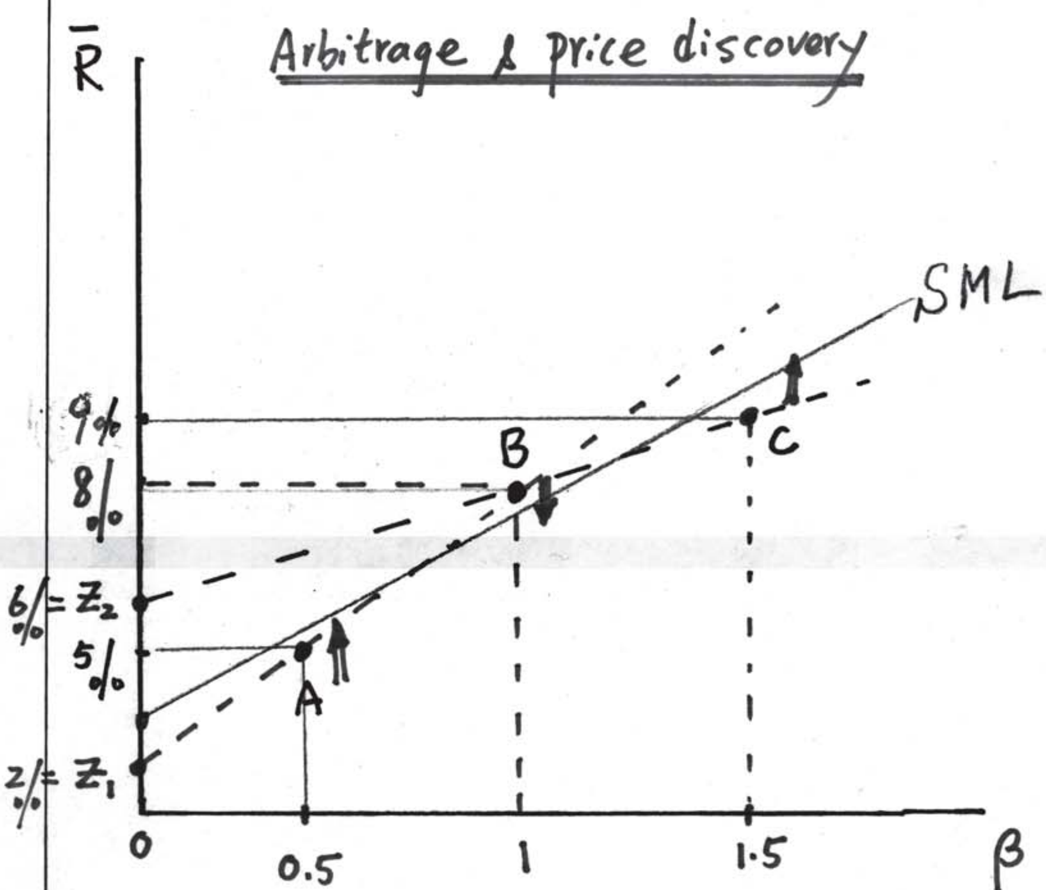
$Price_A = Price_B$

or

Arbitrage Opportunity disappears!

① Short the <u>low</u> price Assets
② Sell them below <u>high</u> price
③ Fast Trading

Arbitrage Pricing Theory



(A) $\beta_{Z_1} = 0 = \omega_A \beta_A + \omega_B \beta_B$

$= 0.5\,\omega_A + (1-\omega_A)\cdot 1$

$\therefore\ \omega_A = 2$ and $\omega_B = -1$

$\bar{R}_{Z_1} = Z_1 = 2(5\%) - 8\% = 2\%$

(B) $\beta_{Z_2} = 0 = \omega_B' \beta_B + \omega_C \beta_C$

$= \omega_B' + (1-\omega_B') \times 1.5$

$\therefore\ \omega_B' = 3$ and $\omega_C = -2$

$\bar{R}_{Z_2} = Z_2 = 3(8\%) - 2(9\%) = 6\%$

$\left\{ \begin{array}{l} \beta_{Z_1} = \beta_{Z_2} = 0 \\ (\bar{R}_{Z_1} = 2\%) < (\bar{R}_{Z_1} = 6\%) \end{array} \right.$

$\Longleftrightarrow P_{Z_1} > P_{Z_2}$

Action: Short Z_1 (Short A & Long B)
Buy Z_2 (Long B & short C)

Arbitrage profit $= P_{Z_1} - P_{Z_2}$

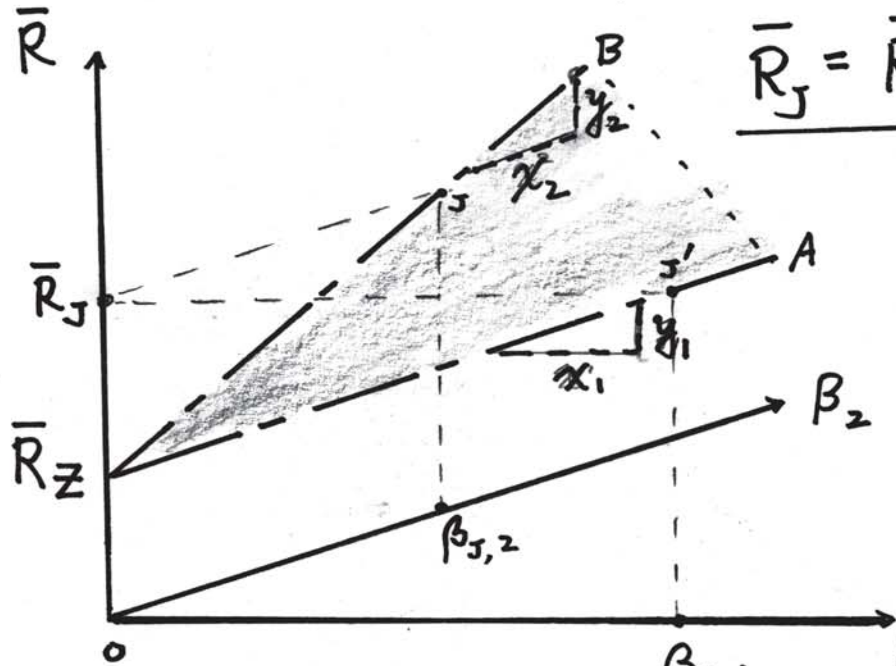
$P_B \uparrow\ \bar{R}_B \downarrow$; $P_A \downarrow\ \bar{R}_A \uparrow$; $P_C \downarrow\ \bar{R}_C \uparrow$

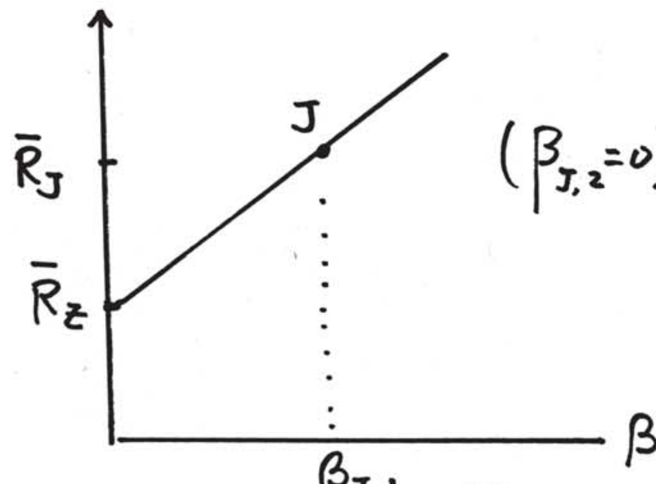
$\Rightarrow$ until SML achieves ← Approches Zero

Arbitrage Pricing Theory

Two Factors:

$$\bar{R}_J = \bar{R}_Z + \beta_{J,1}\lambda_1 + \beta_{J,2}\lambda_2$$

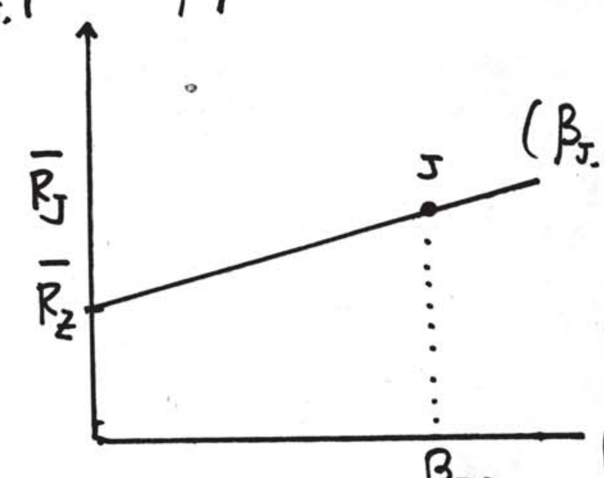




$(\beta_{J,2}=0)$

$$\bar{R}_J = \bar{R}_Z + \beta_{J,1}(\bar{R}_{f1} - \bar{R}_Z)$$

$$\lambda_1 = \frac{y_1}{x_1}$$



$(\beta_{J,1}=0)$

$$\bar{R}_J = \bar{R}_Z + \beta_{J,2}(\bar{R}_{f2} - \bar{R}_Z)$$

$$\lambda_2 = \frac{y_2}{x_2}$$

Arbitrage Pricing Theory

In general, $\bar{R}_J = \bar{R}_z + \sum_{k=1}^{K} \beta_{J,k} \lambda_k = \lambda_0 + \sum_{k=1}^{K} \beta_{J,k} \lambda_k = \sum_{k=0}^{K} \lambda_k \cdot \beta_{J,k}$

λ = factor price = factor risk-premium

<u>Return Genering process</u>:

Consider ① $\tilde{R}_J = \alpha_J + \sum_{k}^{K} \beta_{Jk} \tilde{R}_{fk} + \tilde{\varepsilon}_J$; $\bar{\varepsilon}_J = 0$

② $\bar{R}_J = \alpha_J + \sum \beta_{Jk} \bar{R}_{fk}$

① - ②

$$\tilde{R}_J = \bar{R}_J + \sum \beta_{Jk} (\tilde{R}_{fk} - \bar{R}_{fk}) + \tilde{\varepsilon}_J$$

$$\bar{R}_J = \lambda_0 + \sum \beta_{JK} (\bar{R}_{fK} - \lambda_0) \longleftarrow \text{"from APT"}$$

$$\therefore \quad \tilde{R}_J = \lambda_0 + \sum \beta_{JK} (\tilde{R}_{fK} - \lambda_0) + \tilde{\varepsilon}_J$$

$$= \lambda_0 (1 - \sum \beta_{JK}) + \sum \beta_{JK} \tilde{R}_{fK} + \tilde{\varepsilon}_J$$

Under APT: $\alpha_J = \lambda_0 (1 - \sum \beta_{JK})$