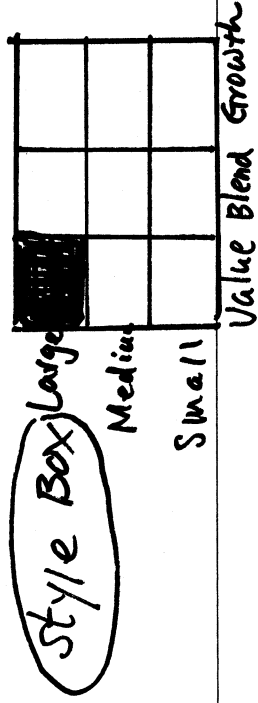


Why security market indexes are important?

- ① Reflect market sentiment
- ② used as a benchmark of manager performance: Information Ratio
- ③ to calculate beta and Alpha
- ④ Model portfolio for index funds (ETFs) Sharp Ratio, α

Types of equity indices:

- ① Broad Market indexes: DJIA, S&P500, Wilshire 5000
- ② Multi-market index: Emerging market index, MSCI World Index
- ③ Market index with fundamental weighting: Global index/GDP
- ④ Sector index: Health Care, Consumer good, financial...
- ⑤ Style index: Large-Value, Small-Cap.....



Security Market Indices : "Price-Weighted Index"

$$\text{Weight} = w_j = \frac{P_j}{\sum P_j}$$

↑ DJIA
e.g.

at $t=2$ (end of $t=1$), Security #1 announces
2-for-1 split

Security	P_0	w_0	P_1	w_1	P_1'	w_1'	P_2	w_2
1	50	49.26%	55	52.38%	27.50	35.48	29	38.67%
2	25	24.63	22	20.95	22	28.39	21	28.00%
3	12.5	12.32	8	7.62	8	10.32	7	9.33%
4	10	9.85	14	13.33	14	18.07	13	17.33%
5	4	3.94	6	5.72	6	7.74	5	6.67%
Total	105	100%	105	100%	77.5	100%	75	100%
Divisor	5		5		3.69		3.69	
Index Value	20.30		21.00		21.00		20.33	

Bias → to the behavior of the highest priced securities.

e.g. SO, Stock split could help to reduce this bias.

Stock	P_0	P_1
1	100	110
2	10	12
Index	$\frac{110}{2} = 55$	$\frac{122}{2} = 61$

Security Market Indices: "Equal-Weighted Index"

Security	P_0	P_1	$(1+R_i)$	P_2	$\frac{1+R_2}{11.5}$
1	10	11	$\frac{11}{10} = 1.1$	11.5	$\frac{11.5}{11} = 1.045$
2	35	33	$\frac{33}{35} = .943$	32	$\frac{32}{33} = .97$
3	100	105	$\frac{105}{100} = 1.05$	54	$\frac{54}{52.5} = 1.029$

Index 200 ? ?

Index₁ $\left\{ \begin{aligned} &200 \left(\frac{1.1 + .943 + 1.05}{3} \right) = 206.20 \leftarrow \text{Arithmetic Average} \\ &200 (1.1 * .943 * 1.05)^{\frac{1}{3}} = 205.78 \leftarrow \text{Geometric Average} \end{aligned} \right.$

Index₂ $\left\{ \begin{aligned} &206.2 \left(\frac{1.045 + .97 + 1.029}{3} \right) = 209.22 \\ &205.78 (1.045 * .97 * 1.029)^{\frac{1}{3}} = 208.69 \end{aligned} \right.$

Biase \Rightarrow to the behavior of the Small price securities in the index

$$\left[R_{\text{Geometric Index}} = \left(\frac{208.69}{205.78} \right) - 1 = 1.41\% \right] < \left[R_{\text{Arithmetic Index}} = \left(\frac{209.22}{206.20} \right) - 1 = 1.47\% \right]$$

Security Market Index: "Value-Weighted Index"

Security	t=0		t=1		t=2	
	Shares Out.	P ₀	Shares	P ₁	Shares	P ₂
1	100 _m	10	100 _m	11	101 _m	10
2	20 _m	35	20 _m	33	21 _m	34
3	10 _m	100	10 _m	105	10 _m	102
Index	200	?	?	?	?	?

$$\text{Index}_1 = 200 \left(\frac{100(11) + 20(33) + 10(105)}{100(10) + 20(35) + 10(100)} \right) = 200 * 1.041 = 208.20$$

$$\text{Index}_2 = 208.20 \left(\frac{101(10) + 21(34) + 10(102)}{100(11) + 20(33) + 10(105)} \right) = 208.20 * 0.9765 = 203.31$$

Bias \implies to the behavior of the highest Capitalization Securities
(e.g. Security 1 & 3)

Security Market Index: How to create an Index Fund?

t=1

Security	Shares	P _i	V _i	w _i
1	100m	11	1100 _m	.39146
2	20m	33	660 _m	.23488
3	10m	105	1050 _m	.37366
			Sum = 2810 _m	1.00000

Fund Capital = \$100,000,000

Security	Allocation	P _i	#shares to Buy
1	39,146,000 / \$11		3,558,727
2	23,488,000 / \$33		711,758
3	\$37,366,000 / \$105		355,867

**

Larger the # of Securities in the index, Larger the Capital is required to reduce tracking errors!